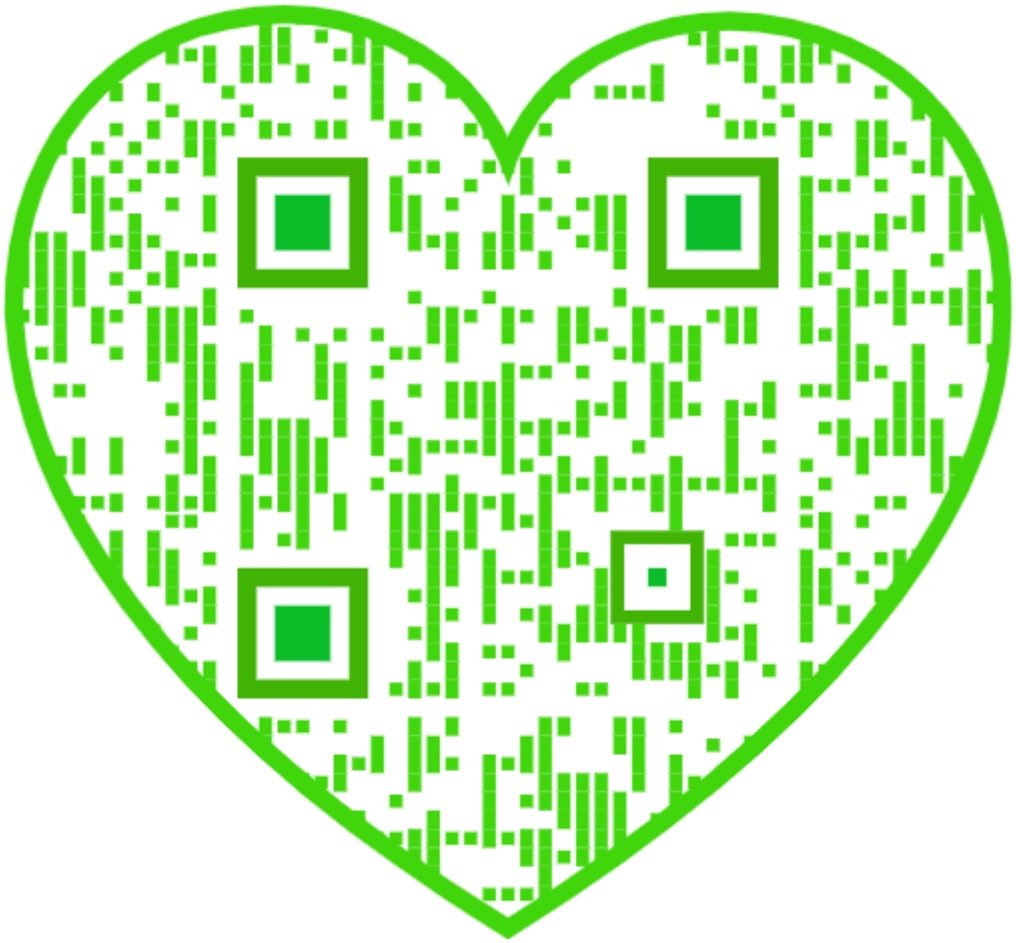

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AGRICULTURAL SURVEYING

A PRACTICAL TREATISE

ON

LAND SURVEYING, LEVELLING, AND SETTING-OUT; AND ON MEASURING AND ESTIMATING QUANTITIES, WEIGHTS, AND VALUES OF MATERIALS, PRODUCE, STOCK, ETC. ETC.

By JOHN SCOTT

EDITOR OF THE "FARMERS' GAZETTE," DUBLIN; LATE PROFESSOR OF AGRICULTURE AND RURAL ECONOMY AT THE ROYAL AGRICULTURAL COLLEGE, CIRENCESTER; AUTHOR OF THE "FARM VALUER," "RENTS AND PURCHASES," ETC.; AND ONE OF THE AUTHORS OF MORTON'S "HANDBOOK OF THE FARM" SERIES

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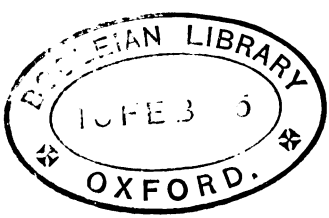

PREFACE.

THIS, the seventh volume of the "Farm Engineering Text-Books," brings the series to a close.

The introductory chapters of this volume treat of the different methods of Land Surveying by the chain; of the instruments used in chain surveying; of noting the measurements; and of plotting and calculating the content. Then follow chapters on Surveying by the Theodolite, and on Levelling. The peculiar system upon which the United States Public Lands are surveyed is explained, in the interest of the large body of immigrant farmers and young men who annually leave our shores for America; and a chapter is devoted to the division and laying out of lands.

The principles of estimating weight, quantity, and values are likewise given as fully as the extent of the work will allow.

Tables of Imperial Weights and Measures are appended, with their Metric equivalents.

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AGRICULTURAL SURVEYING.

CHAPTER I.

LAND SURVEYING.

THE object of land surveying, as usually understood, is the determination of the area and shape of a tract of land.

A survey of this kind usually embraces—

(1.) An examination of a tract as to
Extent,
Contour,
Divisions, &c.

(2.) A plan showing the said features, or some of them.

The duties of a land-surveyor, however, frequently extend beyond making a plan and giving the superficial area; such as disputed boundaries, exchange and division of land, diversion and improvement of roads, as well as the measurement of all kinds of materials and of work performed, &c.

Methods of Land Surveying.—There are but two methods employed in surveying, viz.—

(1.) That by distances and offsets, and

(2.) That by triangles or polygons.

That by distances and offsets is the more simple for

complicated objects, and is suitable to detail operations only, as the position of fences, buildings, &c. It is always based upon the system of triangles.

In surveying by triangles the system of triangles is used. There is a system of surveying by polygons, but it is only a modification of the system of triangles.

Land surveying is further divided into two classes, according to the instruments, &c., employed :—

First, by the chain, or by the chain and cross only ;

Second, by the chain, and the use of the theodolite or other instrument for measuring angles ; and

Third, by trigonometry, which is chiefly performed by the theodolite and logarithmic tables.

The latter is seldom required in agricultural surveying.

The mode of proceeding adopted by land-surveyors varies much with the extent and character of the country to be surveyed ; in fact, under the same circumstances, two different surveyors will perform their work very differently from each other, and with different kinds of instruments, although each may be equally accurate in his results.

And in all the methods of land surveying there are three stages of operations :—

1. *Measuring* certain lines and angles, and *recording* them.
2. *Drawing* them on paper to some suitable scale ; and
3. *Calculating* the contents of the surface surveyed.

CHAPTER II.

INSTRUMENTS USED IN CHAIN SURVEYING.

Gunter's Chain.—This is the instrument most commonly used in land measuring. It is 66 feet, or 4 rods long, and is divided into 100 links, each link being 7·92 inches long.

The reason for having the chain of 66 feet or 100 links is owing to its convenient relation to the standard units, a chain being the $\frac{1}{80}$th part of a mile; the $\frac{1}{10}$th

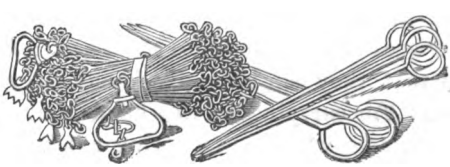


Fig. 1.

part of a furlong, and a square chain being the $\frac{1}{10}$th part of an acre. Ten square chains therefore make one acre, and the computation of areas is thus greatly facilitated.

The links being decimal parts of the chain is also a great advantage, as they may be so written down—10 chains and 80 links, for example, being 10·80 chains; and all the calculations respecting chains and links can then be performed by the common rules of decimal arithmetic.

Every tenth link in the chain is marked by a piece of brass, having one, two, three, or four points, corresponding to the number of tens which it marks, counting from the nearest end of the chain. The middle or fiftieth link is marked by a round piece of brass.

Steel wire is better than iron for surveying chains, owing to its greater strength and stiffness. The rings and eyes which unite the links of a chain are better to be welded than the old way of merely folding the ends. The elliptical form of eye is the best.

The chain should be kept free from bends or crooks in the links, the rings and eyes all clean; and the length should be accurately and regularly tested. It is prudent to test the length of a chain on every occasion of using it, even though the joints are welded.

Arrows.—Ten arrows usually accompany the chain. They are about a foot long, made of stout iron wire sharpened at one end and bent into a ring at the other. Pieces of red and white cloth are tied to their heads, so that they can be easily found in grass, dead leaves, &c. For carrying in the hand they should be strung on a ring which has a spring catch to restrain them.

Signals usually consist of a pole, to be seen at any necessary distance, and they vary in length from 9 feet upwards. They must be planted exactly over the station they are intended to mark, and truly vertical, by looking at it from a little distance, or by holding a plumb line between it and the eye, when it should be looked at from two points as nearly as possible perpendicular to each other.

To render poles conspicuous they should be painted black and white alternately in lengths, or have flags (red or white) attached to them. A red flag is of a

colour not so common as white, but white gives a brighter light than red, and, but for other objects, should be visible at the greatest distance.

The most important point with reference to signals is, the pole should be placed truly vertical, so that if only the top of it can be seen it may cause no error.

How to Chain.—The manner of using the chain should be carefully attended to, so that its length may be always correctly pointed off by the chain-leader, as thus:—

The line having been previously poled out, the surveyor standing at the station point holds one end of the chain, the assistant with the other end in his right hand and the arrows in his left hand, which are transferred one by one into the right hand. On arriving at the extent of the chain, he turns partly round, holding the arrow perpendicular at the end of the chain, looking towards the surveyor, who springs the chain until it is in a straight line with the fore object, then, by motion of the head or hand, directs the leader to move the arrow accordingly, until it is in the proper point of the line and the chain fairly stretched out; the arrow is then to be fixed in the ground and the chain remain at rest until the surveyor has taken all the offsets and remarks necessary, and at his signal proceed on to the next length, and so continuing until the whole ten arrows are fixed or transferred into the surveyor's possession. The leader then proceeds, without any pins, adjusting the chain in the line, which must remain at rest until the surveyor arrives at that end and puts one down, delivering the nine arrows remaining to the leader, each time carefully counting them at every change, and also at the end of every line, to prove that no mistake has occurred by dropping one or by false entry.

The leader should be trained to keep the line by a back object—that is, by placing himself in a line with the arrow last put down and the mark or pole at the station, or some distant object that may accidentally be in line. It saves much time and labour when the chainman is made to keep or pole out a line truly.

Measuring-tapes.—Though the chain is most usually employed for the principal measurements, a tape-line, divided on one side into links and on the other side into feet and inches, is more convenient for taking short lengths, as in measuring offsets, &c. Tapes will all vary in length by the moisture in the air, and hence are not free from errors. Those made of best strong linen wear well, and keep to standard better than any others.

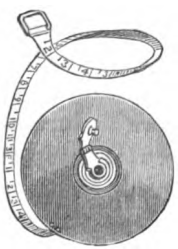


Fig. 2.

Offset-staff.—This is also an important accompaniment to the chain, being convenient for measuring short offsets. It is usually ten links long, painted white, with each link marked by a black painted ring, and the ring numbered 1, 2, 3, &c. The bottom of the rod is shod with an iron spike and the top has a stout open ring, as thus ⚲, to force or draw the chain through the bottom part of a fence.

Cross-staff.—A number of convenient instruments of simple form, known as the cross-staff or the surveyor's cross, are in use for setting out perpendiculars by lines of sight, crossing each other at right angles; and a temporary substitute for them is easily made by sticking a pin in each corner of a square piece of board, and sighting across these in the direction of the line and at right angles to it.

The commonest form of cross-staff is that represented in Fig. 3. It consists of a block of wood (which may be of any shape) having in it two saw-cuts, made very precisely at right angles to each other, and with centre-bit holes made in the bottom of the cuts to assist in finding the objects. This block is fixed on a pointed staff, on which it can turn freely, and which should be precisely 8 links ($63\frac{1}{3}$ inches) long, for the convenience of short measurements.

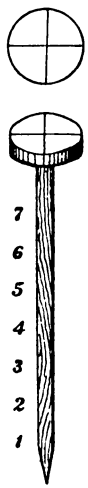


Fig. 3.

To test the accuracy of the instrument, sight through one slip to some point, A (Fig. 4), and place a stake, B, in the line of sight of the other slip. Then turn its head a quarter of the way around, so that the second slip looked through points to A. Then see if the other slip covers B again, as it will if correct.

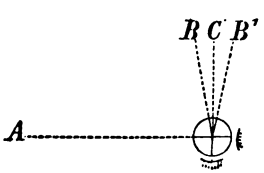


Fig. 4.

If it does not do so, but sights to some other point, as B′, the apparent error is double the real one, for it now points as far to the right of the true point, C, as it did before to its left. The invaluable principle of this test is that it doubles the real error and makes it twice as easy to perceive and correct it.

To use the cross-staff to erect a perpendicular, set it at the point of the line at which a perpendicular is wanted. Turn its head till, on looking through one saw-cut, you can see the ends of the line. Then will the other saw-cut point out the direction of the perpendicular, and thus guide the measurement desired.

There are many improved forms of cross-staffs. Fig. 5 shows one of these. It is made of plain brass, with centre axis and divided circle to take any angle. A compass is sometimes attached to this cross-staff head.

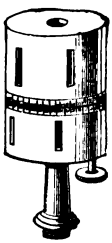


Fig. 5.

Optical Square.—For measuring long offsets and perpendiculars this instrument (Fig. 6) is now very generally used.

Fig. 6.

It is a small circular box containing a strip of looking-glass, from the upper half of which the silvering is removed. This glass is placed so as to make precisely half a right angle with the line of sight, which passes through a slit on one side of the box, and a vertical hair stretched across the opening on the other side, or a mark on the glass.

Another form of the optical square contains two glasses fixed at an angle of 45°, and giving a right angle, or reflecting 90° on both hands.

CHAPTER III.

ON NOTING THE MEASUREMENTS.

Keeping the Field Notes.—In all the methods of surveying, the measurements, together with various incidental observations, are recorded after some established system in what are called *field notes*, and from these the results of the survey are afterwards plotted to a convenient scale.

In chain surveying the most simple method of keeping the field-book is to make a *sketch* of the field, as nearly correct as the unassisted hand and eye can produce, and note down on it the lengths of all the lines, as in Fig. 24. But where many other points require to be noted, such as where fences, or roads, or streams are crossed in the measurement, or any other additional particulars, the sketch would become confused and be likely to lead to mistakes in the subsequent plotting of it. The following is, therefore, the usual method of keeping the field notes. A long narrow book is most convenient for it.

Draw two parallel lines about an inch apart from the bottom to the top of the field-book, as in the margin. This column, or pair of lines, may be considered to represent the measured line split in two, its two parts being thus separated, an inch apart, merely

for convenience, so that the distances measured along the line may be written between these halves.

Hold the book in the direction of the measurement. At the bottom of the page write down the name or number or letter which represents the station at which the survey is to begin.

A "station" is marked with a triangle or circle, as in the margin. The latter is more easily made.

The station from which the measurements are made is usually put on the left of the column, and the station which is measured to is put on the right.

But it is more compact, and avoids interfering with the notes of "offsets" to write the name or number of the station in the column, as in the margin.

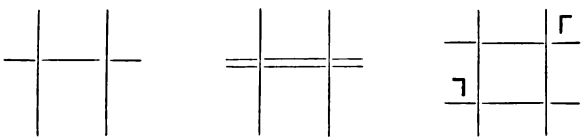
The measurements of different points of a line are written above one another. The numbers all refer to the beginning of the line, and are counted from it.

	⊙	to B	B		B
					400
	562		562		250
					100
From A	⊙		A		A

The end of a measured line is marked by a line drawn across the page above the numbers of the measurements which have been made.

If the chaining does not continue along the adjoining line, but the chain-men go to some other part of the field to begin another measurement, two lines are drawn across the page.

When a line has been measured, the marks ⌜ or ⌝ are made to show whether the following line turns to the right or to the left.



When a mark is left at any point of a line, with the intention of coming back to it again in order to measure to some other point, the place marked is called a *false station*, and is marked in the field-book F.S., or has a line drawn around it to distinguish it, or has a station mark △ placed outside of the column, to the right or left, according to the direction in which the measurement from it is to be made. Examples of these three modes are given in the margin.

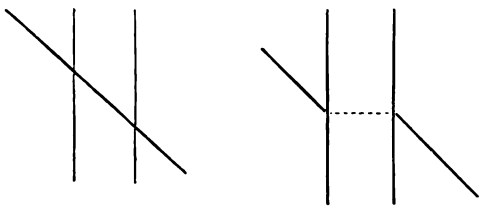
	562	
	200	F.S.
	0	
	562	
	(200)	
	0	
	562	
	200	△
	0	

A false station is named by its position on the line where it belongs; as thus, "200 on 562."

When a gate occurs in a measured line, the distance from the beginning of the line to the side of the gate first reached is the one noted.

When the measured line crosses a fence, brook, or road, &c., they are drawn on the field notes in their true direction as nearly as possible, but not in a continuous

line across the column, as in the first figure in the margin, but as in the second figure, so that the two



parts would form a continuous straight line, if the halves of the "split line" were brought together.

It is convenient to name the lines in the margin as being sides, diagonals, proof-lines, &c.; but in many cases they are denoted by numbers. When two or more lines proceed from the same station, they are distinguished by a smaller figure over, thus 1^2 or 1^3, denoting that a second or third line commenced from that point or station.

Particular attention should be paid in showing the fences, to which field they belong, and where they change, at which point always take an offset. If there is a ditch to the hedge or other fence, the ditch is always the boundary, and is noted in the field-book thus T|L. The line denotes the ditch, the letter T shows the side on which the fence belongs.

When the ditch is next the chain, the offset is taken at right angles from the chain to the edge of the ditch, and when the ditch is outside, the offset is taken to the middle of the fence with the offset staff, and six or seven links added to it, as general allowance for the ditch, about $4\frac{1}{2}$ feet from the middle of the fence.

When there is no ditch on either side, the offset

must then be taken to the centre of the fence, and noted in the field-book thus ———I———.

A paling fence is described thus — — — —, excepting when it is the boundary next a road, in which case there is sometimes a ditch outside. Wire fences are distinguished by a spiral line ~~~~~~~~, and walls or dykes by two parallel lines ═══. Footpaths and roads without fences are shown by small dotted lines . . . . . .

CHAPTER IV.

PLOTTING A CHAIN SURVEY.

THE field work being completed, the figure of the tract surveyed is reproduced upon a diminished scale by what is termed *plotting*, or platting.

A *plot* of a survey is a skeleton or outline map. It is a figure similar to the original, having all its angles equal and its sides proportional. Every inch on it represents a foot, a yard, a rod, a mile, or some other length on the ground, all the measured distances being diminished in exactly the same ratio.

The only instruments absolutely necessary for this are a straight ruler and a pair of dividers or compasses. Others, however, are often convenient, and may be briefly noticed.

Parallels.—The readiest mode of drawing parallel lines is by the aid of a triangular piece of wood and a ruler. Let A B (Fig. 7) be the line to which a parallel is to be drawn, and C the point through which it must pass. Place one side of the triangle against the line, and place the ruler against another side of the triangle. Hold the ruler firm and immovable

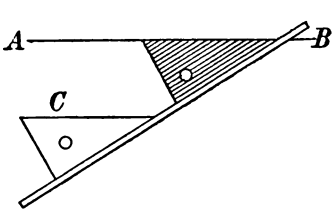


Fig. 7.

and slide the triangle along it till the side of the triangle which had coincided with the given line passes through the given point. This side will then be parallel to that given line, and a line drawn by it will be the line required.

Another easy method of drawing parallels is by means of a T-square, an instrument very valuable for many other purposes. It is nothing but a ruler let into a thicker piece of wood very truly at right angles to it. For this use of it, one side of the "cross-piece" must be even or "flush" with the ruler. To use it, lay it on the paper so that one edge of the ruler coincides with the given line A B. Place another ruler against the cross-piece, hold it firm, and slide the T-square along till its edge passes through the given point C, as shown by the lowest part of the diagram (Fig. 8). Then draw by this edge the desired line parallel to the given line.

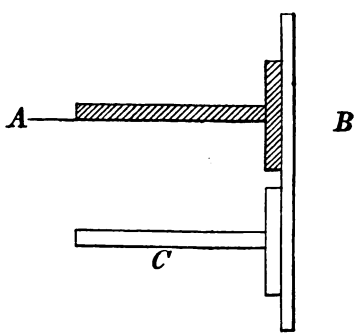


Fig. 8.

Perpendiculars.—These may be drawn by the various problems given in geometry, but more readily by a triangle which has one right angle.

Place the longest side of the triangle on the given line, and place a ruler against a second side of the triangle, as in Fig. 9. Hold the ruler fast, and turn the triangle so as to bring its

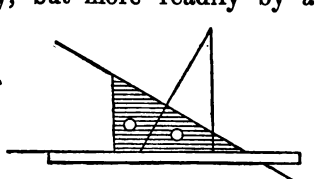


Fig. 9.

third side against the ruler. Then will the long side be perpendicular to the given line. By sliding the triangle along the ruler, it may be used to draw a perpendicular from any point of the line, or from any point to the line.

Perpendiculars are also at times drawn by means of a semicircular protractor.

Drawing to Scale.—The operation of drawing on paper lines whose length shall be a half, a quarter, a tenth, or any other portion of the lines measured on the ground, is called "drawing to scale."

To set off on a line any given distance to any required scale, determine the number of chains or links which each division of the scale of equal parts shall represent. Divide the given distance by this number. The quotient will be the number of equal parts to be taken in the dividers and to be set off.

For example, suppose the scale of equal parts to be a carpenter's common rule, divided into inches and eighths. Let the given distance be 12 chains, which is to be drawn to a scale of 2 chains to an inch. Then 6 inches will be the distance to be set off. If the given distance had been 12 chains and 75 links, the distance to be set off would have been 6 inches and 3-8ths, since each eighth of an inch represents 25 links.

If the desired scale were 3 chains to an inch, each eighth of an inch would represent $37\frac{1}{2}$ links; and the distance of 1,275 links would be represented by thirty-four eighths of an inch, or $4\frac{1}{4}$ inches.

A similar process will give the correct length to be set off for any distance to any scale.

If the scale used had been divided into inches and tenths, as is much the most convenient, the above dis-

tances would have become on the former scale $6\frac{37}{100}$ inches or nearly $6\frac{4}{10}$ inches, and on the latter scale $4\frac{25}{100}$ inches, coming mid-way between the second and third tenth of an inch.

Conversely, to find the real length of a line drawn on paper to any known scale, reverse the preceding operation. Take the length of the line in the dividers, apply it to the scale, and count how many equal parts it includes, multiply their number by the number of chains or links which each represents, and the product will be the divided length of the line on the ground.

Scales.—The choice of the scale to which a plot should be drawn—that is, how many times smaller its lines shall be than those which have been measured in the ground—is determined by several considerations. The chief one is that it shall be just large enough to express clearly all the details which it is desirable to know. A farm survey would require its plot to show every field and building. A State survey would show only the towns, rivers, and leading roads.

Scales are named in various ways. *They should always be expressed fractionally—i.e.*, they should be so named as to indicate what fractional part of the real line measured on the ground the representative line drawn on the paper actually is. It would be better still if the denominator could always be some power of 10, or at least some multiple of 2 or 5, such as $\frac{1}{500}$, $\frac{1}{1000}$, $\frac{1}{2000}$, $\frac{1}{2500}$, &c.

Plots of *farm surveys* are usually named as being so many *chains* to an *inch*.

Maps of *surveys of states* are generally named as being made to a scale of so many *miles* to an *inch*.

Farm Surveys.—If these are of small extent, two

chains to one inch (which is $= 2 \times 66 \times 12 = \frac{1}{1584}$ $= 1 : 1584$) is convenient. A scale of one chain to an inch (1 : 792) is useful for plans of buildings. Three chains to one inch (1 : 2376) is suitable for larger farms or estates. It is the scale prescribed by the (English) Tithe Commissioners for their first-class maps.

The choice of the most suitable scale for the plot of a

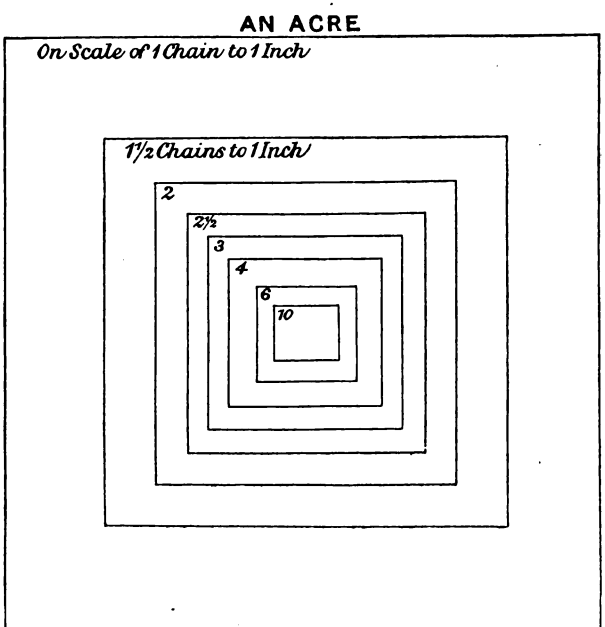


Fig. 10.

farm survey may be facilitated by the preceding figure which shows the actual space occupied by *one acre*, laid out in the form of a square, on maps drawn to the various scales named in the figure.

State Surveys.—On these surveys, smaller surveys, smaller scales, are necessarily employed.

The *Ordnance Survey* of the southern counties of England was plotted on a scale of 2 inches to 1 mile (1 : 31,880), and reduced for publication to that of 1 inch to a mile (1 : 63,300). The scale of 6 inches to a mile (1 : 10,560) was adopted for the northern counties of England and for the southern counties of Scotland.

The scale of the parish plans is 1-2500th of the actual length on the ground, and is equal to 25,344 inches to a mile, which is very approximately equal to one square inch to one acre, the square of 1·0018 inch being equal to one acre.

Whatever scale may be adopted for plotting the survey, it should be drawn on the map, both for convenience and reference, and in order that the contraction and expansion caused by changes in the quantity of moisture in the atmosphere may affect the scale and map alike.

Scale omitted.—It may be required to find the unknown scale to which a given map has been drawn, its superficial contents being known.

Assume any convenient scale, measure the lines of the map by it, and find the contents by the methods to be given in the next chapter, proceeding as if the assumed scale were the true one. Then make this proportion, founded on the geometrical principle that the areas of similar figures are as the square of their corresponding sides.

As the contents found *is* to the given content, *so* is the square of the assumed scale *to* the square of the true scale.

TABLE FOR REDUCING OR ENLARGING PLANS BY THE EIDOGRAPH.

Proportion.	Division in Bars.	Proportion.	Division in Bars.
2 to 1	33·333	3 to 2	20·000
3 to 1	50·000	4 to 3	14·285
4 to 1	60·000	5 to 4	11·111
5 to 1	66·666	6 to 5	9·090
6 to 1	71·428	5 to 2	25·000

Reference Books are essential accompaniments to maps or plans, and are of various kinds. Sometimes they merely contain the names and contents of the fields or other parts or divisions, with the state of culture or condition in which they are; in other cases the soil and subsoil are described; but in the most complete cases each farm is described, together with the history of its occupation or improvement under the following heads:—

Name.
Parish.
Extent.
Boundaries.
How let and managed hitherto.
To whom and for how much let at present.
Description of the farm buildings.
Fences, trees.
Ponds, &c., drain outlets, &c.
Content, soil, subsoil, surface, expanse, &c., of each field.
Proportion of land under timber, coppice, &c.

In addition to such a description as the above, some add in the reference book a separate map of each farm, which renders the whole very comprehensive.

CHAPTER V.

CALCULATING THE CONTENT.

Unit of Content.—The acre is the unit of land measurement. A rood contains 40 perches. A perch is a square rod, otherwise called a perch or pole. A rod is $5\frac{1}{2}$ yds., or $16\frac{1}{2}$ ft. Hence 1 *acre* = 4 roods = 160 perches = 4,840 square yds. = 43,560 square ft. *One square mile* = 5,280 × 5,280 ft. = 640 acres. Since a chain is 66 feet long, a square chain contains 4,356 square feet, and consequently ten square chains make an acre.

In different parts of England the acre varies greatly. The *statute acre* contains 160 square perches of $16\frac{1}{2}$ ft., or 43,560 square ft. The acre of *Devonshire* and *Somersetshire* contains 160 perches of 15 ft., or 36 good square ft.; *Cornwall*, 160 perches of 18 ft., or 51,840 square ft.; *Lancashire*, 160 perches of 21 ft., or 70,560 square ft.; *Cheshire* and *Staffordshire*, 160 perches of 24 ft., or 92,160 square ft.; *Wiltshire*, 120 perches of $16\frac{1}{2}$ ft., or 32,670 square ft.; *Scotland* consists of 10 square *chains, each* of 74 *ft.*, and therefore contains 54,760 square ft.; *Ireland* (same as Lancashire). The Irish chain is 84 ft.

When the content of a piece of land (following any of the methods to be explained presently) is given in square links, as is customary, cut off four figures on

the right (*i.e.* divide by 10,000), to get it into square chains and decimal parts of a chain; cut off the right figure of the *square chains*, and the remaining figures will be *acres*. Multiply the remainder by 4, and the figures if any, outside of the new decimal point will be *roods*. Multiply the remainder by 40, and the outside figure will be *perches*. Thus—

		A.	R.	P.
86·22	square chains =	8	2	20
8·250	,, ,, =	0	3	12
0·8250	,, =	0	0	13

Boundary Lines.—The lines which are to be considered as bounding the land to be surveyed are often very uncertain, unless specified by the title-deeds.

If the boundary be a brook, the middle of it is usually the boundary line. On tide-waters the land is usually considered to extend to low-water mark.

When hedges and ditches are the boundaries of fields, the dividing line is generally the top edge of the ditch farthest from the hedge, both hedge and ditch belonging to the field on the hedge side. This varies, however, with the customs of the locality.

Methods of Calculation.—The various methods employed in calculating the content of a piece of ground may be reduced to four, which may be called *Arithmetical, Geometrical, Instrumental,* and *Trigonometrical.*

First Method.—Arithmetically. From direct measurements of the necessary lines on the ground.

The figures to be calculated by this method may be either the shapes of the fields which are measured, or those into which the fields can be divided by measuring various lines across them.

The familiar rules of mensuration for the principal figures which occur in practice will be now briefly enunciated.

Rectangles.—If the piece of ground be rectangular in shape, its content is found by multiplying its length by its breadth.

Triangles.—When the given quantities are on one side of a triangle, and the perpendicular distance to it from the opposite angle, the content of the triangle is equal to half the product of the side and the perpendicular.

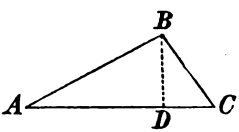


Fig. 11.

When the given quantities are the three sides of the triangle, add together the three sides and divide the sum by 2; from this half sum subtract each of the three sides in turn; multiply together the half sum and the three remainders; take the square root of the product; it is the content required.

Parallelograms, or four-sided figures whose opposite sides are parallel. The content of a parallel equals the product of one of its sides by the perpendicular distance between it and the side parallel to it.

Trapezoids, or four-sided figures two opposite sides of which are parallel. The contents of a trapezoid equals half the products of the sum of the parallel sides by the perpendicular distance between them.

Quadrilaterals, or *Trapeziums,* four-sided figures none of whose sides are parallel.

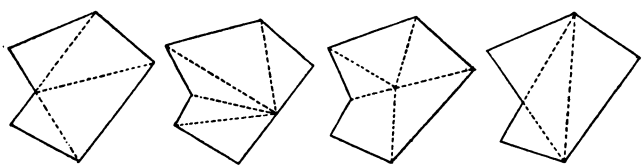
A very gross error often committed as to this figure is to take the average, or half sum of its opposite sides, multiply them together for the area; thus assuming the trapezium to be equivalent to a rectangle with these averages for sides.

In practical surveying it is usual to measure a line across it from corner to corner, thus dividing it into two triangles whose sides are known, and which can therefore be calculated.

Surfaces bounded by irregularly curved lines.—The rules for these will be more appropriately given in connection with the surveys which measure the necessary lines, as will be explained, Part 2, chap. 3.

Second Method. Geometrically.—From measurements of the necessary lines upon the plat or plot.

Division into Triangles.—The plat of a piece of ground having been drawn from the measurements made by any of the methods which will hereafter be explained, lines may be drawn upon the plat so as to divide it into a number of triangles.



Figs. 12, 13, 14, and 15.

Four ways of doing this are shown in the figs. 1. By drawing lines from one corner to the other corner. 2. From a point in one of the sides to the corners. 3. From a point inside of the fig. to the corners. 4. From various corners to other corners. The last method is usually the best. The lines ought to be drawn so as to make the triangles as nearly equilateral as possible.

One side of each of these triangles and the length of the perpendicular let fall upon it being then measured as directed, the content of these triangles can be at once obtained by multiplying their base by their altitude and dividing by two.

The easiest method of getting the perpendicular, without actually drawing it, is to set one point of the dividers at an angle from which a perpendicular is to be let fall, and to open and shut their legs till an arc

described by the other point will just touch the opposite side.

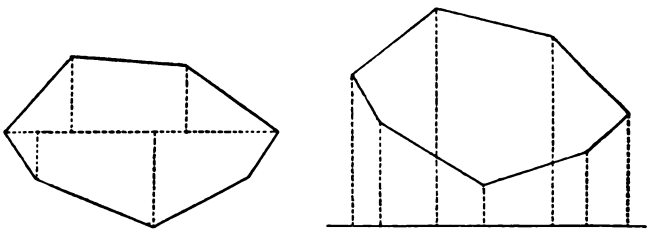
Otherwise, a platting scale may be placed so that the zero point of its edge coincides with the angle, and one of its cross lines coincides with the sides to which a perpendicular is to be drawn. The length of the perpendicular can then at once be read off.

The method of dividing the plat into triangles is the one most commonly employed by surveyors for obtaining the content of a survey, because of the simplicity of the calculations required. Its correctness, however, is dependent on the accuracy of the plat, and on its scale which should be as large as possible. Three chains to an inch is the smallest scale allowed by the Tithe Commissioners for plats from which the content is to be determined.

Some surveyors measure the perpendicular of the triangles by a scale half of that to which the plat is made. Thus, if the scale of the plat be 2 chains to an inch, the perpendiculars are measured with a scale of 1 chain to an inch. The product of the base by the perpendicular thus measured gives the area of the triangle at once, without its requiring to be divided by 2.

Division into Trapezoids.—A line may be drawn across the field, as in Fig. 16, and perpendiculars drawn to it. The field will thus be divided into trapezoids (except a triangle at each end), and their contents can be calculated. Otherwise, a line may be drawn outside the figure, and perpendiculars to it be drawn from such angle, as in Fig. 17. In that case the difference between the trapezoids formed by lines drawn to the outer angle of the fig. and those drawn to the inner angles will be the content. This method

is very advantageously applied to surveys by the compass.



Figs. 16 and 17.

Division into Squares.—Two sets of parallel lines at right angles to each other, one chain apart (to the scale of the flat) may be drawn over the plat, so as to divide it into squares, as in Fig. 18. The number of

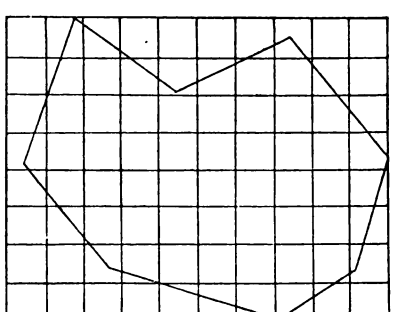


Fig. 18.

squares which fall within the plat represent so many square chains, and the triangles and trapezoids which fall outside of these may then be calculated and added to the entire square chains which have been counted.

Instead of drawing the parallel lines on the plat,

they may better be drawn on a piece of transparent "tracing-paper," which is simply laid upon the plat and the squares counted as before. The same paper will answer for any number of plats drawn to the same scale. This method is a valuable and easy check on the results of other calculations.

To calculate the fractional parts, prepare a piece of tracing-paper by drawing on it one square of the same size as a square of the plat, and sub-dividing it by two sets of 10 parallels at right angles to each other, into hundreds. This will measure the fractions remaining from the former measurement as nearly as can be desired.

Divisions into Parallelograms.—Draw a series of parallel lines across the plat at equal distances depending on the scale. Thus, for a plat made to a scale of 2 chains to an inch, the distance between the parallel should be $2\frac{1}{2}$ inches, 3 chains to an inch should be $1\frac{1}{3}$ inch, 4 chains to an inch should be $\frac{5}{8}$ inch. 5 chains to an inch should be $\frac{4}{10}$ inch; and for any scale make the distance between the parallels that fraction of an inch which would be expressed by 10 divided by the square of the number of chains to the inch.

Then apply a common inch scale, divided on the edge into tenths, to these parallels, and every inch in length of the spaces included between each pair of them will be an acre, and every tenth of an inch will be a square chain.

To measure the triangles at the ends of the strips between the parallels, prepare a piece of stout tracing-paper of a width equal to the width between the parallel, and draw a line through its middle longitudinally. Apply it to the oblique line at the end of the space between two parallels and it will bisect the

line, and thus reduce the triangle to an equivalent rectangle as at A in Fig. 19. When an angle occurs between two parallels, as at B in the fig., the fractional part may be measured by any of the preceding methods.

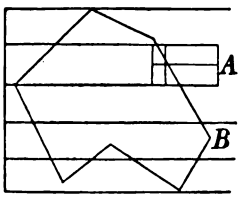


Fig. 19.

Addition of Widths.—When the lines of a plat are very irregularly curved, as in Fig. 20, draw across it a number of equi-distant lines as near together as the

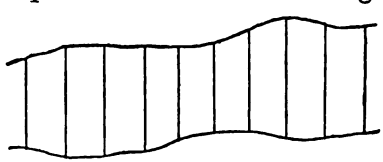


Fig. 20.

case may seem to require. Take a straight-edged piece of paper, and apply one edge of it to the middle of the first space, and mark its length from the end; apply the same edge to the next space, bringing the mark just made to one end, and making another mark at the end of the additional length; so go on, adding the length of each space to the previous ones. When all have been thus measured, the total length, multiplied by the uniform width, will give the contents.

Third Method. Instrumentally.—By performing certain instrumental operations on the flat.

Any plain figure bounded by straight lines may be reduced to a single triangle which shall have the same content. This can be done by any instrument for drawing parallel lines.

Special Instruments. — Computing and calculating scales, &c., are also used for finding the content.

Fourth Method. Trigonometrically.—By calculating from the observed angles of the boundaries of the piece of ground the lengths of the lines needed for calculating the content.

This method is employed for surveys made with angular instruments, as the compass, &c., in order to obtain the content of the land surveyed, without the necessity of previously plotting it, thus avoiding both that trouble and the inaccuracy of any calculations founded upon it. It is, therefore, the most accurate method, but will be more appropriately explained under the head of compass surveying.

Logarithms.—The logarithm of a number is the exponent of the power to which it is necessary to raise a fixed number to produce the given number. The fixed number is called the *base.* Thus, in the equation $10^3 = 1000$, 3 is the log. of 1000, the base being 10. Any positive number except 1 may be taken as a base, and for each base there is a corresponding *system of logarithms.* There is therefore an infinite number of systems of logarithms, but only two of them are in general use—the *Napierian* system, whose base is 2·718281828, mostly employed in the higher branches of analysis and in scientific investigations; and the *Common* system, whose base is 10, used in practical computations, where they (the logs.) serve to convert the operations of multiplication and division into the simpler ones of addition and subtraction. (We adopt the latter.)

In trigonometric computations the use of logarithms is almost indispensable.

Computations by means of logarithms are made in accordance with the following principles:—

(1.) The log. of the product of any number of factors is equal to the sum of the logs. of the factors.

(2.) The log. of a quotient is equal to the log. of the dividend diminished by that of the divisor.

(3.) The log. of any power of a quantity is equal to the log. of the quantity multiplied by the exponent of the power; and

(4.) The log. of any root of a quantity is equal to the log. of the quantity divided by the index of the root.

In applying these principles, the logs. needed are taken from tables called tables of logarithms.

The manner of arranging the tables, and also the manner of using them, will be best learned from the explanations which precede each collection of tables.

The following tables will be found useful :—

CHAINS INTO FEET.

Chains.	Feet.	Chains.	Feet.
0·01	0·66	1·00	66·
0·02	1·32	2	132
0·03	1·98	3	198
0·04	2·64	4	264
0·05	3·30	5	330
0·06	3·96	6	396
0·07	4·62	7	462
0·08	5·28	8	528
0·09	5·94	9	594
0·10	6·60	10	660
0·20	13·20	20	1320
0·30	19·80	30	1928
0·40	26·40	40	2640
0·50	33·00	50	3300
0·60	39·60	60	3960
0·70	46·20	70	4620
0·80	52·80	80	5280
0·90	59·40	90	5940
1·00	66·00	100	6600

FEET INTO LINKS.

Feet.	Links.	Feet.	Links.
0·10	0·15	10	15·2
0·20	0·30	15	22·7
0·25	0·38	20	30·3
0·30	0·45	25	37·9
0·40	0·60	30	45·4
0·50	0·76	33	50·0
0·60	0·91	35	53·0
0·70	1·06	40	60·6
0·75	1·13	45	68·2
0·80	1·21	50	75·8
0·90	1·36	55	83·3
1·00	1·52	60	90·9
2·	3·0	65	98·5
3·	4·5	70	106·1
4·	6·1	75	113·6
5·	7·6	80	121·2
6·	9·1	85	128·8
7·	10·6	90	136·4
8·	12·1	95	143·9
9·	13·6	100	151·5

To *reduce links to feet*, subtract from the number of links as many units as it contains hundreds; multiply the remainder by 2 and divide by 3.

To *reduce feet to links*, add to the given number half of itself, and add one for each hundred (more exactly, for each 99) in the sum.

To convert decimal fractions of an acre into roods and perches, multiply the decimal first by 4 and then by 40, preserving the same number of decimals in the product.

Examples—

	Acres.		Acres.
	633·357		527·013
	4		4
	1·428		·052
	40		40
A. R. P.		A. R. P.	
633 1 17	17·120	527 0 2	2·080

TABLE

FOR CONVERTING DECIMAL PARTS OF AN ACRE INTO ROODS AND PERCHES.

Perch.	0 Rood.	One Rood.	Two Roods	Three Roods.	Perch.	0 Rood.	One Rood.	Two Roods.	Three Roods.
0	·000	·250	·500	·750	21	·131	·381	·631	·881
1	·006	·256	·506	·756	22	·137	·387	·637	·887
2	·012	·262	·512	·762	23	·144	·394	·644	·894
3	·019	·269	·519	·769	24	·150	·400	·650	·900
4	·025	·275	·525	·775	25	·156	·406	·656	·906
5	·031	·281	·531	·781	26	·162	·412	·662	·912
6	·037	·287	·537	·787	27	·169	·419	·669	·919
7	·044	·294	·544	·794	28	·175	·425	·675	·925
8	·050	·300	·550	·800	29	·181	·431	·681	·931
9	·056	·306	·556	·806	30	·187	·437	·687	·937
10	·062	·312	·562	·812	31	·194	·444	·694	·944
11	·069	·319	·569	·819	32	·200	·450	·700	·950
12	·075	·325	·575	·825	33	·206	·456	·706	·956
13	·081	·331	·581	·831	34	·212	·462	·712	·962
14	·087	·337	·587	·837	35	·219	·469	·719	·969
15	·094	·344	·594	·844	36	·225	·475	·725	·975
16	·100	·350	·600	·850	37	·231	·481	·731	·981
17	·106	·356	·606	·856	38	·237	·487	·737	·987
18	·112	·362	·612	·862	39	·244	·494	·744	·994
19	·119	·369	·619	·869	40	·250	·500	·750	1·000
20	·125	·375	·625	·875					

CHAPTER VI.

METHODS OF CHAIN SURVEYING.

Stations.—In surveying there is a preliminary process of choosing suitable stations on the ground—such as will be well seen from each other; and straight lines may be necessary between them.

The principal station-lines may not be sufficient for all the work of the survey, but subordinate lines and stations are easily made out, forming a network of triangles.

The principal stations must be chosen before the survey is begun; they should be well seen from each other, and give a clear straight line for measurement between them.

The subordinate stations and lines are chosen as the work proceeds, with a view to running the lines near the fences, &c.

Marks, or Bench Marks, are made for reference to positions or levels. The most commonly used are wooden stakes or pegs. Care should be taken to place them truly vertical; and if the head of the pin is large, the precise point should be marked upon it by driving in a nail.

Measuring Straight Lines.—The lines or distances to be measured may be either actual or visual. Actual lines are such as really exist on the surface of the land

to be surveyed, either bounding it or crossing it, such as fences, ditches, roads, streams, &c. Visual lines are imaginary lines of sight, either temporarily measured on the ground, or simply indicated by stakes at their extremities. If long they are "ranged out," by methods to be hereafter explained.

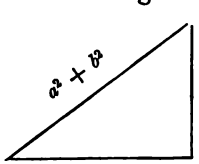
Lines are usually measured with chains, tapes, or rods, divided into yards, feet, links, or some other unit of measurement.

Crooked lines are determined by means of perpendicular offsets measured from different points along the straight line, run as nearly coincident to the crooked line as may be.

When the boundary is irregular, the area of the residual portions which necessarily lie beyond the limits of the triangulation are estimated by the method of taking offsets, which will be fully described in a succeeding chapter of this book.

Triangulation.—Right angles may be set off on the ground by means of the chain.

The setting out of right-angled triangles by the chain, and the reduction of slopes to horizontals, are done by Euclid I. and 47.



In all triangulation (whether by angles or by measured lines), care should be taken to avoid ill-conditioned triangles, or such as have very acute or very obtuse angles. Generally, the surveyor should avoid angles greater than 120° or less than 30°.

An error on the ground generally amounts to an increased error on the paper. The limit of uncertainty, or the probable area of error, will be least when the angles are right angles.

Proof Lines, measured from the corner of each triangle to the opposite side, serve to rectify the other measures of the triangle, and if perpendicular to the side afford a convenient means of calculating upon the ground the area of the triangle.

Measurement of Angles.—The angle made by any two lines—that is, the difference of their directions—is measured by various instruments, consisting essentially of a circle divided into equal parts, with plain sights, or telescopes, to indicate the directions of the two lines. But angles in the field are also determined by the chain. This is done by measuring a *tie-line* from a measured point on one side to a measured point on the other side. By this means the boundaries of a tract may be determined when it cannot be conveniently measured off in triangles.

Horizontal Measurement.—All ground, however inclined or uneven its surface may be, should be measured horizontally and as if brought down to a horizontal plane, so that the surface of a hill, thus measured, would give the same content as the level base on which it may be supposed to stand.

Take, for example, a level field surrounded by a fence in the shape of a perfect square whose side is 10 chains. The area of the field is 10 acres. Through a convulsion of nature a large mound is thrown up in the centre, two or three hundred feet in height, leaving the fences undisturbed. The area of the surface now contained is obviously greater than before, although a surveyor would take no notice of the fact, but would make the area 10 acres, as originally. The necessity for adopting this system of surveying will be seen on considering the diagram.*

* *Knowledge*, November 30th, 1883.

A B C D is a square whose side is 10 chains in length; the mound is shown by dotted lines in the centre. On measuring the line E F along the surface of the ground its length is found to be greater than 10 chains, and this length set off from E would extend to F, distorting

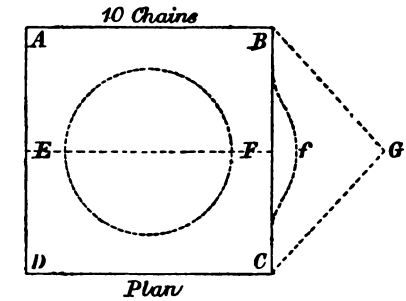


Fig. 21.

the fence, which we know has not changed its position, and bringing it nearer the point C, which we also know is not the case. The necessity for reducing the measurements to a horizontal datum is at once seen.

This is necessary for geometrical reasons, as otherwise in mapping a survey every hilly field or tract would overlap its real boundary.

Horizontal measurement is also justified by the fact that no more houses can be built on a hill than could be on its flat base; and that no more trees, corn, or other

plants which shoot up vertically, can grow on it, as is represented by the vertical lines in the section, Fig. 21.

Hilly land is, therefore, always bought and sold in accordance with horizontal measurement.

Chaining on Slopes.—All the distances employed in land surveying must be measured horizontally, or on a level. In chaining uneven or sloping ground, therefore, it is necessary to make certain allowances or corrections. The chain may be held horizontally by the eye, or the slope may be taken by the theodolite, and horizontal distance calculated from the slope of the ground.

When the angle of the slope is measured, the calculation may also be made by a table already prepared. In the following table, the first column contains the angle which the surface of the ground makes with the horizon; the second column contains its slope named by the ratio of the perpendicular to the base; and the third the connection in links for each chain, measured on the slope, *i.e.* the difference between the hypothenuse, which is the distance measured, and the horizontal base, which is the distance desired.

TABLE FOR CHAINING ON SLOPES.

Angle.	Slope.	Correction in Links.	Angle.	Slope.	Correction in Links.
3°	1 in 19	0·14	13°	1 in 4½	2·56
4°	1 „ 14	0·24	14°	1 „ 4	2·97
5°	1 „ 11½	0·38	15°	1 „ 4	3·41
6°	1 „ 9½	0·55	16°	1 „ 3¾	3·87
7°	1 „ 8	0·75	17°	1 „ 3½	4·37
8°	1 „ 7	0·97	18°	1 „ 3¼	4·89
9°	1 „ 6½	1·23	19°	1 „ 3	5·45
10°	1 „ 6	1·53	20°	1 „ 2¾	6·03
11°	1 „ 5½	1·84	25°	1 „ 2	9·37
12°	1 „ 4¾	2·19	30°	1 „ 1¾	13·40

Surveying by Diagonals.

Surveying by Diagonals is an application of the *first method* of determining the position of a point already shown, to which the student should again refer. Each corner of the field or farm which is to be surveyed is "determined" by measuring its distances from two other points. The field is then "platted" by repeating this process on paper for each corner, in a contrary order, and the "content" is obtained by some of the methods already explained.

The lines which are measured in order to determine the corners of the field are usually *diagonals* of the irregular polygon which is to be surveyed. They, therefore, divide up this polygon into triangles, whence this method of surveying is sometimes called "chain triangulation."

A few examples will make the principle and practice perfectly clear.

A Three-sided Field. Field Work.—Measure the three sides A B, B C, and C A. Measure also, as a proof-line, the distance from one of the corners, as C, to some point in the opposite side, as D, at which a mark should have been left when measuring from A to B, at a known distance from A. A stick or thing with a slit in its top to receive a piece of paper, with the distance from A marked on it, is the most convenient mark.

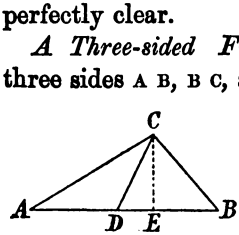


Fig. 22.

Platting.—Choose a suitable scale, then draw a line equal in length, on the chosen scale, to one of the sides; A B, for example. Take in the compasses the length of another side, as A C, to the same scale, and with one

leg in A as a centre describe another arc, intersecting the first arc in a point which will be the third corner, C. Draw the lines A C and B C, and A B C will be the plat or miniature copy of the field surveyed.

Instead of describing to acres to get the point C, two pairs of compasses may be conveniently used. Open them to the lengths respectively of the last two sides. Put one foot of each at the ends of the first side, and bring their other feet together, and their point of meeting will mark the desired third point of the triangle.

To "prove" the accuracy of the work, fix the point D by setting off from A the proper distance, and measure the length of the line D C. If its length on the plat correspond to its measurement on the ground, the work is correct.

Calculation.—The content of the field may now be found, either from the three sides, or more easily though not so accurately by measuring on the plat the length of the perpendicular C E, let fall from any angle to the opposite side, and taking half the product of these two lines.

Example 1.—Fig. 22 in the plat, on a scale of 2 chains to an inch, of a field of which the side A B is 200 links, B C is 100 links, and A C is 150 links. Its content is 0·726 of a square chain, or 0 acres, 0 rods, 12 poles. If the perpendicular A D be accurately measured, it will be found to be 72½ links. Half the product of this perpendicular by the base will be found to give the same content.

Example 2.—The three sides of a triangular field are respectively 89·38, 54·09, and 45·98. Required its content. *Answer*, 100 acres 0 roods 10 poles.

The field notes of the triangular field plotted in

Fig. 49 are given below, according to both the methods mentioned in the preceding article.

In the field-notes in the column on the right hand it is not absolutely necessary to repeat the B and C.

Proof-line.	From D	89 F. S.	to C
Side.	From C	150 ⊙	to A ⅂
Side.	From B	100	to C
Side.	From A	200 80 ⊙	to B F. S.

Proof-line.	From	C 89 (80)	on 200
Side.	⅂	A 150 C	
Side.	⅂	C 100 B	
Side.		B 200 (80) A	

A Four-sided Field. Field Work.—Measure the four sides. Measure also a diagonal as A C, thus dividing the four-sided field into two triangles. Measure also the other diagonal as a "proof-line."

Platting.—Draw a line, as A C, equal in length to the diagonal, to any scale; on each side of it construct

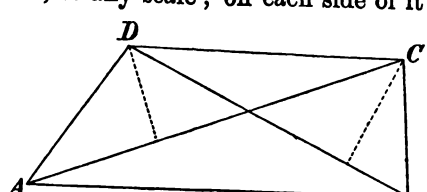


Fig. 23.

a triangle with the sides of the field, as directed in the preceding article.

To prove the accuracy of the work, measure on the plat the length of the "proof-lines," B D, and if it agrees with the length of the same line measured on the ground, the field work and platting are both proved to be correct.

Calculation.—Find the content of each triangle separately, as in the preceding case, and add them together; or, more briefly, multiply either diagonal (the large one is preferable) by the sum of the two perpendiculars, and divide the product by two. Otherwise, reduce the four-sided figure to one triangle, or use any of the methods of the preceding chapter.

Example 1.—In the field drawn in Fig. 23, on a scale of 3 chains to the inch, A B = 588 links, B C = 210 links, C D = 430, D A = 274, the diagonal A C = 626, and the proof diagonal B D = 500. The total content will be 1 acre 0 rods 17 poles.

Example 2.—The side of a four-sided field are A C = 12·41, B C = 5·86, C D = 8·25, D A = 4·24; the diagonal B L = 11·55, and the proof-line A C = 11·04. Required the content. *Ans.*, 4 acres 2 rods 38 poles.

A many-sided Field. Field Work.—Measure all the sides of the field. Measure also diagonals enough to divide the field into triangles, of which there will always be two less than the number of sides. Choose such diagonals as will divide the field into triangles as nearly equilateral as possible. Measure also one or more diagonals for "proof-lines."

Platting.—Begin with any diagonal and plat one triangle. Plat a second triangle adjoining the first one. Plat another adjacent triangle, and so proceed till all are laid down in their proper places. Measure the proof-lines as in the last article.

Calculation.—Proceed to calculate the content of the

figure precisely as directed for the four-sided field, measuring the perpendiculars and calculating the content of each triangle in turn; or taking in pairs those on opposite sides of the same diagonal; or using some of the other methods which have been explained.

Example 1.—The six-sided field chosen in Fig. 24 has the length of its lines, in chains and links, written

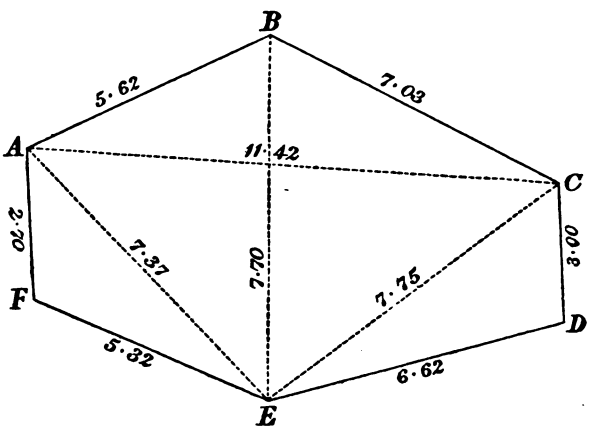


Fig. 24.

upon them, and is divided into four triangles, by three diagonals. The diagonal B E is a "proof-line." The fig. is drawn to a scale of 4 chains to the inch. The content of the field is 5 acres 3 rods 22 poles.

Example 2.—In a five-sided field the length of the sides are as follows: A D = 2·69
B C = 1·22
C D = 2·32
D E = 3·55
E A = 3·23

The diagonals are A D = 4·81
and . . . . . B D = 3·33 Required the content.

A field may be divided up into triangles, not only by measuring diagonals as in the last figure, but by any of the methods shown in the four figures. The one which we have been employing corresponds to the last of those figures.

Still another may be used when the angles cannot be seen from one another, or from any point within. Take three or more convenient points within the field, and measure from them to the corners, and thus form different sets of triangles.

The field-notes of the survey plotted in Fig. 24 are given below. They begin at the bottom of the left-hand column.

Side	F	
	532	
	300	Gate.
	E	⌐
Side.	E	
	662	Brook.
	400	
	D	⌐
Side.	D	
	300	
	270	Road.
	211	
	80	
	C	⌐
Side.	C	
	703	
	150	Gate.
	B	⌐
Side.	B	
	562	
	A	

Proof-line.	E	
	770	
	B	
Diagonal.	A	
	1142	
⌐	C	
Diagonal.	C	
	775	
	480	Road.
	420	
⌐	E	
Diagonal.	E	
	737	
	280	Road
	210	
	A	
Side.	A	
	270	
	130	Road.
	80	
	F	⌐

Surveying by Tie-lines.—This is a modification of the method explained in the previous article. It frequently happens that it is impossible to measure the diagonals of a field of many sides, in consequence of obstacles to measurements, such as woods, water houses, &c. In such cases "tie-lines" (so called because they *tie* the sides together) are employed as substitutes for diagonals. Thus, in the four-sided field shown in the figure, the diagonals cannot be measured because of woods intervening. As a substitute, measure off from any convenient corner of the field, as B, any distance, B E, B F, along the sides of the field. Measure all the sides of the field as usual.

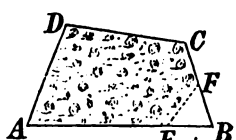


Fig. 25.

To plot this field, construct the triangle B E F, produce the sides B E and B F, till they become respectively equal to B A and B C, as measured on the ground. Then with A and C as centres, and with radii equal A D and C D, describe arcs whose intersection will be D, the remaining corner of the field.

It thus appears that one tie-line is sufficient to determine a four-sided field; two a five-sided field, and so on. But as a check on errors, it is better to measure a tie-line for each angle, and the agreement, in the plot, of all the measurements will prove the accuracy of the whole work.

The tie-lines should be as long as possible, as the nearer the corner the tie-line is drawn the more it magnifies inaccuracies in the side lines.

A tie-line may also be employed as a *proof-line* in the place of a diagonal, and tested in the same manner. If any angle of a field is re-entering, as

at B, Fig. 26, measure a tie-line across the salient angle, A B C.

Chain Angles.—It is convenient, though not necessary, to measure equal distances along the sides; B E, B F, in Fig. 25, and B A, B C in Fig. 26. "Chain angles" are thus formed.

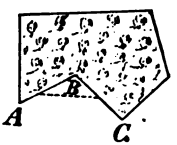


Fig. 26.

Inaccessible Areas.—The method of tie-lines can be applied to measuring fields which cannot be entered. Thus, in the fig., A B C D is an inaccessible wooded field of four sides. To survey it, measure all the sides, and at any corner, as D, measure any distance D E—the line D A produced. Measure also another distance, D F, in the line of C D produced. Measure the tie-line E F, and the figure can be platted as in the case of the field of Fig. 24, the sides of the triangle being produced in the contrary direction.

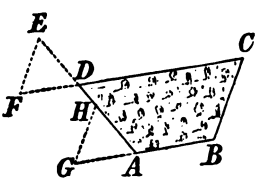


Fig. 27.

The same end would be obtained by prolonging only one side, as shown at the angle A of the same figure, and measuring A G, A H, and G H. It is better in both cases to tie all the angles in a similar manner.

This method may be applied to a figure of any number of sides by prolonging as many of them as are necessary, all of them if possible.

Surveying by Perpendiculars.—The method of surveying by perpendiculars is founded on the second method of determining the position of a point explained. It is applied in two ways, either to make a complete survey by "*Diagonals* and *Perpendiculars*," or

to measuring a crooked boundary by "offsets." Each will be considered in turn.

The best methods of getting perpendiculars on the ground must, however, be first explained.

To set out Perpendiculars.

Surveyor's Cross.—The simplest instrument for this purpose is the *surveyor's cross* or cross-staff, shown in Fig. 3.

Optical Square.—The most convenient and accurate instrument for taking perpendiculars is, however, the *optical square,* Fig. 6.

Chain Perpendiculars.—Perpendiculars may be set out with the chain alone by a variety of methods, these methods generally consist in performing on the ground the operations executed on paper in practical geometry, the chain being used in place of the compasses to describe the necessary arcs.

These operations, however, are less often used for the method of surveying now to be explained than for overcoming obstacles to measurement.

Perpendiculars to any line are readily laid out with a chain, as carpenters and masons draw right angles by what they call the 6, 8, and 10 rule, the popular application of the principle of the square of the hypothenuse being equal to the sum of the squares of the other two sides. The method is to measure from the point where the perpendicular meets the line, either along the line or along the perpendicular, a distance equal to 6 units of any kind, and then upon the other of these lines a distance of 8 units. The two lines are perpendicular to each other when the two termini are just 10 units apart. Convenient distances for this measurement

might be 3, 4, and 5 rods or chains, or any similar multiples of these numbers, as 21, 28, and 35. Other trigonometrical methods readily suggest themselves.

Diagonals and Perpendiculars.

We have seen that plats of surveys made with the *chain* alone have their contents most easily determined by measuring on the flat the perpendiculars of each of the triangles into which the diagonals measured on the ground have divided the field. In the method of *surveying by diagonals and perpendiculars*, now to be explained, the perpendiculars are *measured on the ground.* The content of the field can therefore be found at once (by adding together the half products of each perpendicular by the diagonal on which it is let fall), without the necessity of previously making a plat, or if necessary the sides of the field. This is, therefore, the most rapid and easy method of surveying *when the content alone is required*, and *is particularly applicable to the measurement of ground occupied by crops*, for the purpose of determining the number of bushels grown to the acre, the amount to be paid for mowing by the acre, &c.

A Three-sided Field.—Measure the longest side, A B, and the perpendicular, C D, let fall upon it from the opposite angle, C.

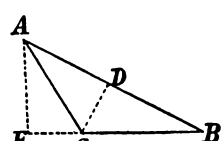


Fig. 28.

Then the content is equal to half the product of the side by the perpendicular.

If obstacles prevent this, find the point where a perpendicular let fall from the angle, as A, to the opposite side produced, as B C, would meet it, as at E in the figure.

A Four-sided Field.—Measure the diagonal A C. Leave marks at the points on the diagonal at which perpen-

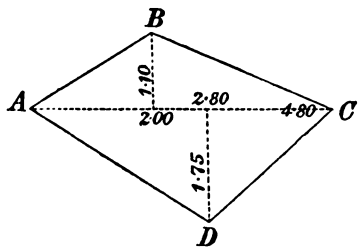


Fig. 29.

diculars from B and from D would meet it, finding these points by trial.

The best marks at these "false stations" have been described.

Example 1.—Required the content of the field, Fig. 29. *Answer*, 0 acres 2 rods 29 poles.

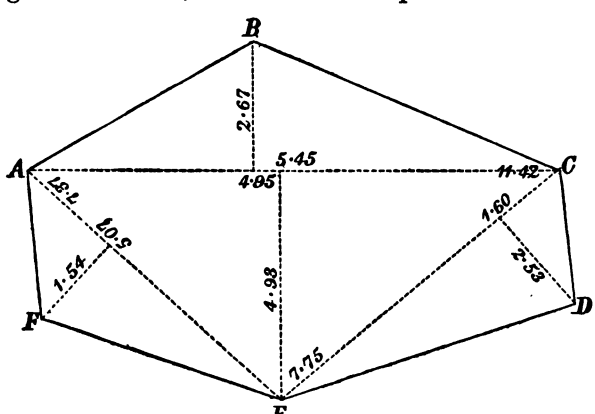


Fig. 30.

The field may be plotted from these measurements if desired, but with more liability to inaccuracy than in

the *first method*, in which *the sides are measured.* The plat of the figure is 3 chains to an inch.

Example 2.—Calculation :—(Fig. 29.)

	Sq. Links.
A B C = ½ × 480 × 110 =	26400
A D C = ½ × 480 × 175 =	42000
Sq. Chains . .	6·8400
Acres	0·684

It is still easier to take the two triangles together, multiplying the diagonal by the sum of the perpendicular and dividing by 2.

A Many-sided Field.—Fig. 30 and the accompanying field notes represent the field which was surveyed by the *first* method and plotted in Fig. 24.

	1·54	to F.
From 5·07 on 7·37 .	F. S.	
	2·53	to D.
From 1·60 on 7·75 .	F. S.	
	4·93	to E.
From 5·45 on 11·42 .	F. S	
	2·67	to B.
From 4·95 on 11·42 .	F. S.	
	7·37	to A.
F. S.	5·07	
From E.	⊙	⌜
	7·75	to E.
F. S.	1·60	
From C.	⊙	⌜
	11·42	
	5·45	F. S.
F. S.	4·95	
From A.	⊙	

Example 1. *Calculations.*—The content of the triangles may be expressed thus:—

						Sq. Links.
A B C	=	½	×	1142 × 267	=	152457
A E C	=	½	×	1142 × 493	=	281503
C D E	=	½	×	775 × 253	=	98037
A E F	=	½	×	737 × 154	=	56749
				Sq. Chains . .		58·8746
				Acres		5·88746

The first two triangles might have been taken together, as in the previous field.

Content calculated from the perpendiculars will generally vary slightly from that obtained by measuring on the plat.

Offsets are short perpendiculars, measured from a straight line to the angle of a crooked or zigzag line

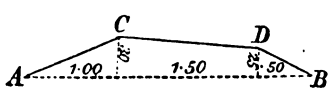
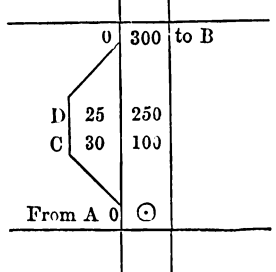


Fig. 31.

near which the straight line runs. Thus, let A B C D be a crooked fence bounding one side of a field. Chain



along the straight line A B, which runs from one end of the fence to the other, and when opposite each corner

note the distance from the beginning, or the point A, and also measure and note the perpendicular distance of each corner, C and D, from the line.

A more extended example, with a little different notation, is given in Fig. 32.

	B	
	1500	0
	1250	20
0	1000	0
30	750	
50	500	
40	250	
	0	
	A	

Fig. 32.

Fig. 32.

In the figure, which is on a scale of 8 chains to an inch for the distance along the line, the breadth of the offsets are exaggerated to four times their true proportional dimensions.

The offsets may generally be taken with sufficient accuracy by measuring them as nearly as possible at right angles to the base line as the eye can estimate.

They may be measured, if short, with an offset-staff, a light rod 10 or 15 links in length, and divided accordingly; or, if they be long, with a tape. They are generally but a few links in length. A chain's length should be the extreme limit. When the cross-staff is in use, its divided length of 8 links renders the offset-staff needless.

The offsets are to be taken to every angle of the

piece or other crooked line; that is, to every point where it changes its direction.

Platting.—The most rapid method of platting the offsets is by the use of a *platting scale* and an *offset scale*, which is a short scale divided on its edges like a platting scale, but having its zero in the middle, as in Fig. 33.

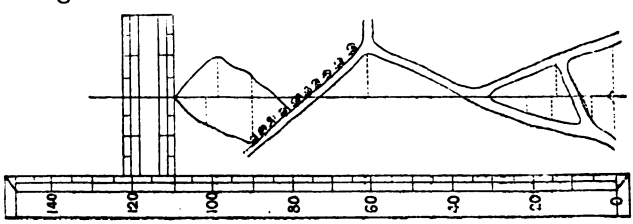


Fig. 33.

The platting scale is placed parallel to the line, with its zero point opposite the beginning of the line. The offset scale is slid along the platting scale till its edge comes to a distance on the latter at which an offset had been taken, the length of which is marked off with a needle point from the offset scale. This is then slid on to the next distance, and the operation is repeated. If one person reads off the field notes and another plats, the operation will be greatly facilitated. The points thus obtained are joined by straight lines, and a miniature copy of the curved line is thus obtained, all the operations of the platting being merely repetitions of the measurements made on the ground.

Calculating Content.—When the crooked line determined by offsets is the boundary of a field, the content enclosed between it and the straight line surveyed must be determined, that it may be added to or subtracted from the content of the field bounded by straight lines. There are various methods of effecting this.

The area enclosed between the straight lines and the crooked lines is divided up by the offsets into *triangles* and *trapezoids*, the content of which may be calculated separately and then added together. *Example* 1. The content of Fig. 32 will therefore be 1500 + 4125 + = 6250 square links = 0·625 square chains. *Example* 2. The content of plat, Fig. 32, will in like manner be found to be on the left of the straight line 30·000 square links, and on its right 5·000 square links.

When the offsets have been taken at equal distances, the content may be more easily obtained by adding together half of the first and of the last offset, and all the intermediate ones, and multiplying the sum by one of the equal distances between the offsets. This rule is merely an abbreviation of the preceding one.

Thus, in Fig. 32 the distances being equal, the content of the offset on the left of the straight line will be 120 × 250 = 30,000 square links, and on the right side 20 × 250 = 5,000 square links—the same result as before.

When the line determined by the offsets is a curved line, "Simpson's Rule" gives the content more accurately. To employ it, an *even* number of *equal* distances must have been measured in the part to be calculated; then add together the *first* and *last* offsets, four times the sum of the *even* offsets (*i.e.* the 2nd, 4th, 6th, &c.), *twice* the *sum* of the odd offsets (*i.e.* the 3rd, 5th, 7th, &c.), *not* including the first and last. Multiply the sum by one of the equal distances between the offsets and divide by 3. The quotient will be the area.

Example 1. The offsets from a straight line to a curved fence were 8, 9, 11, 15, 16, 14, 9 links, at equal distances of 5 links. What was the content included

between the curved fence and the straight line? *Answer*, 371·666.

Many *erroneous rules* for calculating offsets have been given, such as—(*a*). To divide the sum of all the offsets by one less than their number, and multiply the quotient by the whole length of the straight line; or what is the same thing, to multiply the sum of all the offsets by the common distance between them.

(*b*). To divide the sum of all the offsets by their number, and then to multiply the quotient by the straight line.

Reducing to one triangle the many-sided figures which are formed by the offsets is the method of calculation sometimes adopted.

Equalizing, or *giving* and *taking*, is an approximate mode of calculation much used by practical surveyors. A crooked line, determined by offsets, having been platted, a straight line is drawn on the plat across the crooked line, leaving as much space outside of the

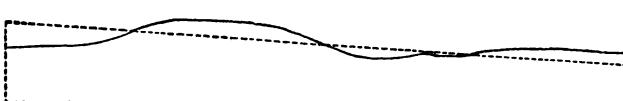


Fig. 34.

crooked line as inside of it, as nearly as can be estimated by the eye, *equalizing* it, or *giving* and *taking* equal portions. The straight line is best determined by laying across the irregular outline the straight edge of a piece of transparent horn, or tracing-paper, or glass, or a fine thread or horsehair. In practical hands this method is sufficiently accurate in most cases.

Surveying by Diagonals, Tie-lines, and Perpendiculars combined.

All the above methods of surveying, and that of per-

pendiculars particularly, in the form of offsets, are frequently required in the same survey.

The method of *diagonals* should be the leading one. In some parts of the survey obstacles to the measurement of diagonals may require the use of *tie-lines*; and if the fences are crooked, straight lines are to be measured near them, and their crooks determined by *offsets*.

Offsets are necessary additions to almost every other method of surveying. In the smallest field surveyed by diagonals, unless all the fences are perfectly straight lines, their bends must be determined by offsets. The plot (scale 1 chain to an inch) and field notes of such a case are given below. A sufficient number of the sides, diagonals, and proof-lines, to prove the work, should be plotted before plotting the offsets.

		C	
Side.	0	360	
	6	315	
	10	275	
	5	215	
	0	150	0
		115	10
		80	5
		65	8
		B	0 ⌜
		B	
Side.	0	125	
	11	90	
	23	62	
	12	22	
	0	A	

Proof-Line.		B	
		340	
		D	
Diagonal.		C	
		310	
		A	⌜
		A	
Side.	0	248	
	11	180	
	0	105	0
		65	5
		D	0 ⌜
		D	
Side.	0	135	
	15	110	
	13	90	
	0	50	0
		30	9
		C	0 ⌜

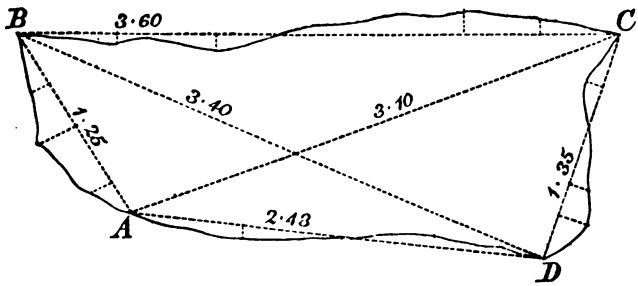


Fig. 35.

Inaccessible Areas.—A combination of *offsets* and *tie-lines* supplies an easy method of surveying an inaccessible area, such as a pond, swamp, forest, block of houses, &c., as appears from the Fig. 36, in which the

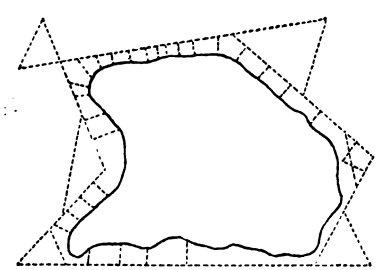


Fig. 36.

external boundary lines are taken at will and measured, and tied by the "tie-lines" measured between those lines, prolonged when necessary, while offsets from them determine the irregularities of the actual boundaries of the pond, &c.

These offsets or insets and their content is, of course, to be subtracted from the content of the principal figure.

Even a circular field might thus be approximately measured from the outside.

A great variety of expedients are adopted for overcoming natural obstacles and determining the extent and shape of inaccessible objects, systems of triangles being in such cases formed outside of and around such objects.

In surveying buildings and enclosures which cannot be passed through, it is done by means of rectangles, as shown in Fig. 37.

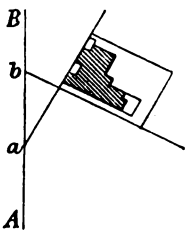


Fig. 37.

CHAPTER VII.

SURVEYING WITH THE THEODOLITE.

A MORE common system of surveying (than with the chain alone) is that in which instruments for taking angles are employed in connection with the chain. A graduated horizontal circle, with a straight edge called an alidade turning upon its central point, which may be conveniently sighted along, furnishes the means of ascertaining the regular distances of two lines, the instruments being set at their intersection, and the alidade pointed in the direction of one and then of the other. This involves the principle of the engineering *transit* or of the *theodolite*.

With these instruments angles can be determined with great accuracy, especially when the observations are repeated by reversing the instrument and taking the mean, each including the reading of both verniers.

A tract of almost any dimensions is accurately surveyed by measuring the angles at its corners, or the correction of the work is proved when the product of all the interior angles is found equal to the product of two right angles, or 180°, by the number of sides of the tract less two; or if the instruments be used by the method called *traversing*,* "or surveying by the back

* *Traversing* is a combination of linear and angular measurement.

angle" (which consists in noting the angle which each successive line makes, not with the preceding line, but with the first line observed, which is hence called the meridian of the survey), then the reading, on getting round to the last station, and looking back to the first line, should be 360° or 0°.

Transit Theodolite.—The transit theodolite is the most perfect instrument used in surveying, and measures at the same time both the horizontal angles between two objects observed with it, and the angles of elevation of these points from the point of observation.

Fig. 39 represents Stanley's 5-inch transit theodolite, with pure achromatic telescope, erect and invert-

Traversing surveys are made by measuring the lengths of the bases, and the angles they make with each other.

As checks:—

(1.) The exterior angles of the polygon should amount to 360°.*

(2.) Fix a point near the centre and divide the polygon into triangles, and measure the angles this point makes with the base lines.

(3.) By prolonging two adjacent sides so as to form one large triangle, and measuring the angles B, C, D, and E, and also P and Q if desired.

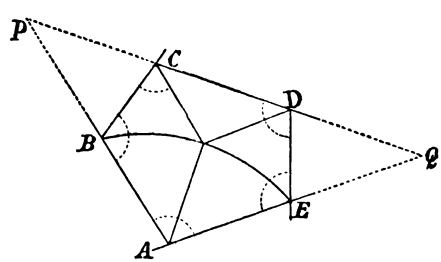


Fig. 38.

The above method by polygons is suitable for the boundaries of fields, estates, &c.

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* Σ External $\angle^s = 360°$.
Internal angles of any polygon $= (n - 2)\ 180° + x$.
$n =$ number of sides of polygon.
x calculated from area of figure.

ing eye-pieces, vertical and horizontal circles divided in silver to 30′, two verniers and two microscopes to each circle, clamping and tangent screw motions, mounted on a mahogany tripod stand.

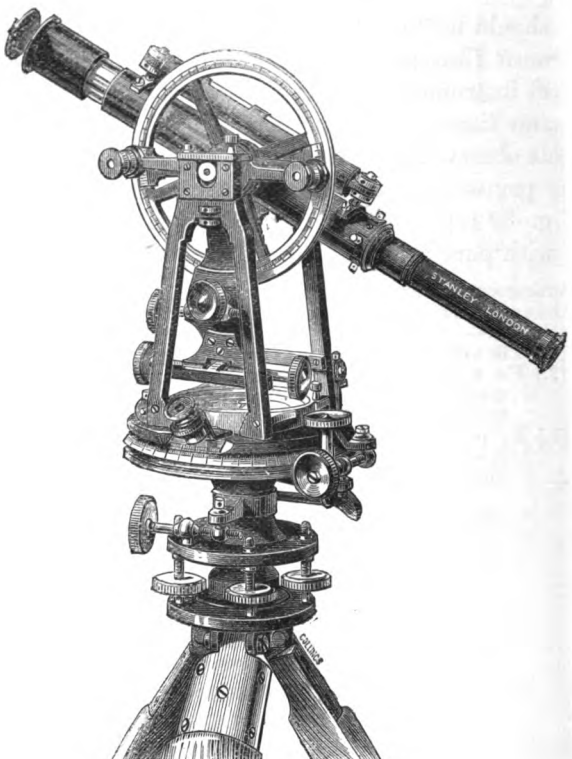


Fig. 39.

On the vernier of some theodolites are three indices at angles of 120°, instead of two indices at 180°. Two indices correct for eccentricity; three indices correct for eccentricity and ellipticity.

Theodolite Adjustments.—The adjustments for the theodolite are :—

Temporary—

(1.) Vertical axis at ⊙.

(2.) Level the instrument, or place vertical axis truly vertical; and

(3.) Adjust the prevent parallax, or bring the foci of the glasses to the cross-wires.

The first and second adjustments require to be made every time the instrument is set up; the third may require to be made at every observation.

Permanent—

(1.) Adjust the line of collimation.

(2.) Adjust the level.

(3.) Ascertain index-error of vertical circle; and

(4.) Adjust horizontal axis exactly perpendicular to vertical axis.

Prismatic Compass.—A compass may be employed in filling up the interior details of a large survey with the transit.

Surveying by the compass is done by taking the bearings of the measured bases from the magnetic meridian. The magnetic needle, wherever the instrument is set, establishes the meridian line, and from this, the sights of the instrument having been turned to any other line, the angle of divergence is read on the graduated circle around the compass box. It is rapid, but not accurate; but it may be depended upon as safe from great errors, such as 10° or any whole number.

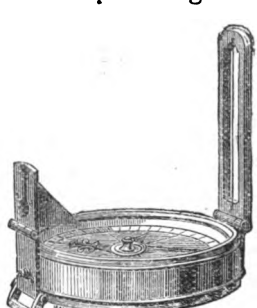


Fig. 40.

The instrument, as in Fig. 40, is usually furnished with sights for the more accurate noting of the angles.

Box-Sextant.—A box-sextant will be found a more valuable auxiliary in filling up the details of a survey.

To save time in surveying, angles should be measured directly with the sextant. This is done by means of a level table upon which the sextant is laid; and poles are used, or a plumb line, &c., to enable the observer to bring the line of sight of the objects to the same horizontal plane.

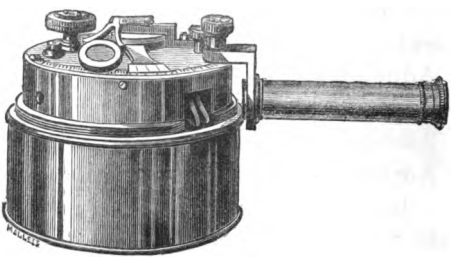


Fig. 41.

One of Stanley's box-sextants, divided in silver, with telescope and supplementary arc, is shown in Fig. 41.

Adjustments—

(1.) Index glass	.	Index glass ⊥ to plane of instrument.
(2.) Horizon glass	.	Horizon glass ⊥ to plane of instrument.
(3.) Index error	.	Ascertain the index error.
(4.) Collimation	.	Line of collimation of telescope ∥ to plane of instrument.

Correction for "index error" in adjusting the sextant may be necessary for two causes, but it is generally

only the first of these that requires attention, the cause of the second happening very rarely in surveys with such instruments.

(1.) Deviation of the index from its correct position.

(2.) The object looked at may be so near the observer that the rays coming from it may not be sensibly parallel, as they would be at a distance.

TABLE FOR ASCERTAINING HEIGHTS AND DISTANCES BY THE BOX-SEXTANT.

Mul.	Angle.	Angle.	Div.
	Degs.	Degs.	
1	45·00	45·00	1
2	63·26	26·34	2
3	71·34	18·26	3
4	75·58	14·02	4
5	78·41	11·19	5
6	80·32	9·28	6
8	82·52	7·08	8
10	84·17	5·43	10

In land measuring on a large scale, a theodolite is invariably used for the measurement of angles. A base line is first chosen and carefully measured, and from each extremity, which is marked by some object visible from a considerable distance, the angle between the other extremity and an arbitrarily chosen and convenient point is measured. This may be done directly, or, as is more usual, the geographical bearings of the new point with respect to the other points are measured, the orientation of the base line itself being already known. Thus the base and contiguous angles of the triangle are given, and from these the other sides and area can be easily calculated. Each of the sides is now taken as a new base line, and new triangles are constructed upon them by arbitrarily choosing new ver-

tices; and thus, by the simple observation of the necessary angles and the careful measurement of one base line, a large tract of country is triangulated or surveyed.

To tell the accuracy of the observations and to fix the limits of error, the last side, whose calculated value depends upon *all* the observations leading up to it, is measured directly as the original base line was.

When the triangulation extends over a whole country, corrections must be applied to the value calculated, because of the sphericity of the earth. The triangles are not plain but spherical, and the problem is therefore really one of spherical trigonometry.

The *details of surveys* are necessarily modified according to the extent of the area, character of the ground, &c. With the transit or compass, the bounding lines may be all followed out, and the angles they make with each other determined and their lengths measured by the chain; the points of crossing of roads, brooks, fences, &c., measured, and the bearings of these objects taken; and increased accuracy may be given to the work by measuring *diagonal or proof-lines*, as in chain surveying.

Additional checks are furnished by taking at each station the bearings of square-marked objects, which, when the work is plotted, should severally fall at the point of intersection of the lines directed toward these objects from the several stations.

Sometimes a tract may be surveyed from a measured base line, either a line within or without it, or one of the boundary lines, by placing the compass successively at each end of this line and taking the bearings of each corner; or, without a compass, the work may be very conveniently performed with appropriate correctness by

plane *table method,* provided no angles are taken less than 30° nor larger than 150°.

A drawing-board covered with paper is set up at one end of a measured base line, and a ruler furnished with upright sights at each end, exactly over the drawing edge, is set with this edge against a fine needle stuck up in the board, and is then directed successively towards the covers of the tract to be surveyed and any other prominent objects, towards which from the needle lines are to be drawn on the paper. One of these lines should be in the direction of the measured line.

The instrument is then taken to the other end of the measured line, the needle is removed along the last line named on the board a distance corresponding, according to the scale adopted, to that of the measured line on the ground, and the board is so placed as to make the line toward the former station.

The ruler is then again pointed to the same object, and lines are drawn toward each from the new position of the needle. Their intersection with the former lines designate the places of these objects on the plane.

The *plane* table is used in various other ways, as by moving it from one corner to the next, and placing it at each so that the last line drawn coincides with that in the ground. From any central point also radiating may be measured to the corners, and the distances measured and marked off to the proper scale.

Rivers, brooks, and roads are surveyed by measuring a succession of lines following their several courses, and taking offsets from the sides of the line.

Protractor.—To accompany the theodolite, &c., there must be provided a protractor, to plot the angles that are taken by the instrument.

The circular protractor with vernier and arm, as in Fig. 42, is the one now commonly used.

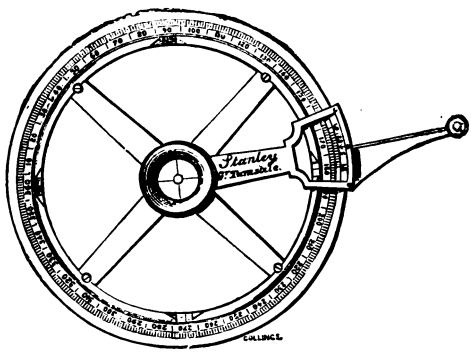


Fig. 42.

To Measure an Angle by the Protractor.—From the centre of the protractor at the point or angle A, and the edge along the line A B, extend the lines sufficiently

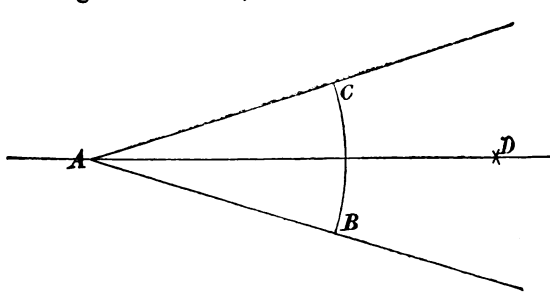


Fig. 43.

to read where it cuts the outer edge of the protractor, and the number of degrees and minutes it reads from B to C will be the measure of the angle required.

CHAPTER VIII.

LEVELLING.

LEVELLING is the art of determining the difference of the heights of two or more points, or of finding the comparative heights of different points of the earth's surface. It is that branch of geodesy which treats of the measurement of heights either (1) *absolute*, when referring to the sea level, or (2) *relative*, between any two distant places on the earth's surface.

Methods.—There are three principal and independent methods in use.

The *first* and most accurate method—that of direct levelling—employs the levelling instrument, and depends on the property of fluids when at rest to present their surfaces at right angles to the direction of gravity.

The *second*, or trigonometrical method, employs the theodolite, and depends on the angular measure of elevation, in combination with the known distance of the object, and having regard to the effect of atmospheric refraction. This is the only method applicable in case one or both stations are inaccessible.

The *third* or barometric method depends on the law of the decrease of pressure of the atmosphere with an increase of altitude. It is the least accurate method of the three, and is of no value for determining small differences of level at any two or more points, though

very serviceable in ascertaining approximately the altitude of a station, mountain, &c., above sea-level.

Level Lines and Surfaces.—A *level surface* is one that is concentric with the surface of the ocean; that is, with the surface the ocean would have if the globe were entirely covered with water.

Any line drawn on a level surface is a *level line.*

A surface of *apparent level* at any point is a plane drawn tangent to the surface of true level at that point. Any line drawn on a surface of apparent level is a line of apparent level.

The lines indicated by our levelling instruments are *lines of apparent level,* but we may deduce from them lines of *true level* by making suitable corrections for curvature and refraction. With short distances, however, such corrections are unnecessary.

Curvature.—The level line given by an instrument is tangent to the surface of the earth. Therefore the line of true level is always the line of apparent level. In Fig. 44, A B represents the line of apparent level, and A C the line of true level. B C is the correction for the earth's curvature.

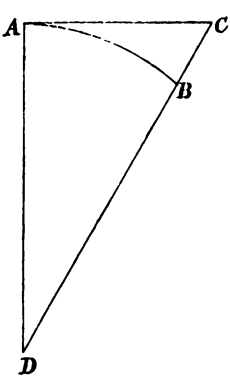


Fig. 44.

$AB^2 = BC \times (BC + {}_2CD)$. But B C being very small compared with the diameter of the earth, may be dropped from the quantity in the parenthesis, and we have—

$$BC = \frac{AB^2}{{}_2CD}$$

that is, the correction equals the square of the distance divided by the diameter of the earth.

The difference of height for a distance of

$$1 \text{ mile} = \frac{1}{7916} = \frac{5280 \times 12}{7916} = 8 \text{ inches.}$$

This varies as the square of the distance. The effect, if neglected, is to make distant objects appear lower than they really are.

The effect is destroyed by setting the instrument mid-way between the two points.

Refraction.—Rays of light coming through the air are curved downwards. The effect is to make objects look higher than they really are. It amounts to about one-seventh that of curvature, and it operates in a contrary direction.

Correction.—Correction for curvature is, therefore, always to be subtracted—

$$= \frac{x^2}{2R} \qquad \begin{array}{l} x = \text{distance in feet.} \\ R \text{ earth's radius.} \end{array}$$

Refraction is to be added—

$$= \frac{x^2}{12R}$$

Curvature and refraction combined is to be subtracted from staff-readings—

$$= \text{on an average } \frac{5}{6} \cdot \frac{d^2}{2R} = \frac{5}{6} \cdot \frac{\text{distance}'^2}{41{,}778{,}000'}.$$

Levelling Instruments.—The instruments used in levelling are of two classes. Those of the first class are used to point out or indicate a line or surface of apparent level, and are technically termed *levels;* those of the second class are used to measure the distances of this line or surface of apparent level above the points whose difference of level is to be determined, and these are called *levelling rods.*

Levels.—These are constructed on one of three principles—

(1.) A line of apparent level is perpendicular to a plumb line freely suspended.

(2.) A line of apparent level is tangent to the free surface of a liquid in equilibrium; and

(3.) A ray of light which is perpendicular to a vertical mirror is a line of apparent level.

(1.) The levels used by bricklayers, carpenters, &c., afford an example of the method of applying the first principle. In its simplest form, this kind of level consists of a T-shaped frame, the line corresponding to the top of the T being perfectly straight and at right angles to a second line drawn through the middle of the stem of the T. A plumb-line is attached to some point of the second line, and when the instrument is held so that the plumb-line corresponds to this second line, the first line is a line of apparent level. The cross line of the T may be turned downwards, as is usually the case when used by mechanics, or it may be turned upwards, in which case, if supported on a suitable stand, it can be used for the rougher kinds of field levelling.

(2.) The ordinary Dumpy level (Fig. 45) affords an illustration of the second principle. It consists essentially of a telescope mounted on a tripod stand. The tripod itself is attached to a solid bar called the limb, which turns about an axis at right angles to it, and so arranged that the axis may be made vertical by the aid of levelling screws. Attached to the telescope is a ring compass and a delicate spirit-level. The latter, when in adjustment, is parallel to the line of collimation of the telescope, which is indicated by two cross hairs mounted on an adjustable diaphragm placed in the common focus of the field lens and eye-piece.

The parts of the instrument are so constructed that they may be brought into accurate adjustment—that is, with proper relative positions. When the instrument

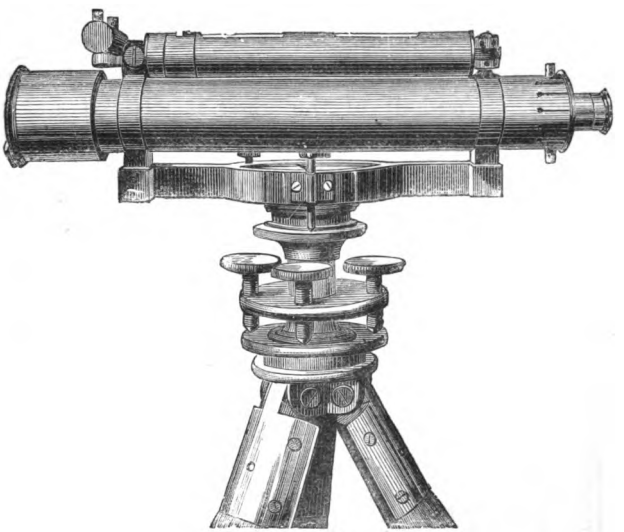


Fig. 45.

is adjusted, the attached level is parallel to the line of collimation of the telescope, and both are perpendicular to the axis of the limb, that is, the line that remains fixed when the limb is turned in *azimuth*.

The necessary adjustments of level are—

Temporary.—(1.) Level; (2.) Correct parallax.

Permanent.—(1.) Line of collimation || spirit level; (2.) Traverse.

To use the instrument thus adjusted we plant the tripod firmly in the ground, and by means of the levelling screws bring the level in such a position that the bubble will remain in the middle of the tube during an entire revolution in *azimuth*. The axis of the limb is then

vertical, and consequently the line of collimation of the telescope in all its positions is a line of apparent level.

(3.) Levels constructed on the third principle are called *reflecting levels*. One form of this class of levels consists of a plate of glass suspended from a ring and weighted so that the plane of the glass shall always be vertical. One-half of the glass is silvered and the other half unsilvered, the line of division between the two portions being vertical. A line is ruled across the middle of the plate, perpendicular to the one last mentioned, and is consequently horizontal. To use the instrument, it is held by the ring, and raised or lowered until the observer sees the image of his eye reflected from the ruled horizontal line on the silvered portion; the plane through the eye in that position, and the line of the unsilvered portion, is a plane of apparent level. Instruments of this kind are convenient for contouring in topographical surveys, but they are not very accurate.

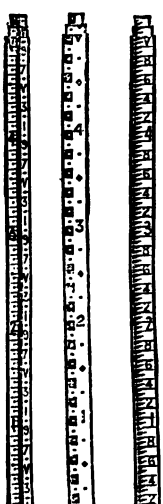


Fig. 46.

Levelling Rods.—These are rods of wood graduated to feet and decimals of a foot, the lines of division being numbered from below upwards; the 0 of the scale is at the bottom of the rod.

The one mostly used consists of a staff of hard wood in three slides or sections, and has the end capped with metal. It is made in length varying from 10 or 14 to 18 feet. The rod may be graduated in different ways; three patterns are shown in Fig. 46. A is a pattern of Sopwith's levelling rod; B is Rogers's field-pattern; and C is the Stanley pattern.

Another form of rod is now much used. It consists of a simple rod without a sliding vane, the divisions and numbers being so distinct as to make them easily read.

In levelling, the rod requires to be held at changes of slope, streams, banks, lines of communication, bench marks, &c.

A plummet is not necessary to set the staff perpendicular, for if the staff be moved backwards and forward, the true reading will be the smallest.

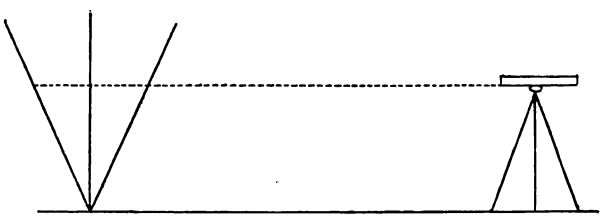


Fig. 47.

Sights.—A pair of sights, or readings, are required to make one complete observation. The first is called the back sight, and the second the fore sight.

The names back sight (B.S.) and Fore Sight (F.S.) do not necessarily mean sights taken looking backwards or forward, though they generally are so for turning points, but the first sight taken after setting up the instrument is a B.S. or + (plus) sight, and all following ones, taken before removing the instrument, are F.S., or — (minus) sights.

All but the first and last points sighted are called *intermediate points*, or "intermediates." The last point sighted to before moving the instrument is called a "turning point" or *changing point.*

Levels may be marked on the ground along a line of road or railway, &c., at distances of from 66 to 300 ft.

The principal reasons for taking sights at short dis-

tances is the uncertainty of the refracting action of the air. They should not be more than 10 chains, or 660 feet.

The only reason for a long sight is when great accuracy is not needed; then we may take the distances as far as we can read the staff, and reduce for curvature and refraction.

Most errors of observation increase as the distance simply.

Errors of refraction increase as the square of the distance merely.

But there are always errors which are constant, and do not vary with the distance.

The greater the distance the greater the effect, and therefore the more need that the instrument be planted exactly half way between fore and back sights.

Direct Levelling.—The levelling instrument is set up mid-way between any two consecutive stations, A and B, on the line of levels, and after its adjustment the

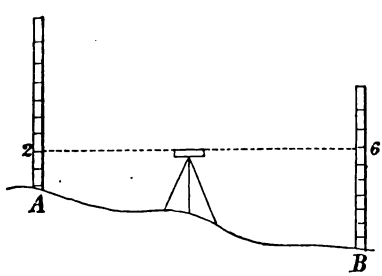


Fig. 48.

readings of the staves placed over the stations are necessarily taken. The line of sight having been made horizontal, the difference in the readings equals the difference of level of the two points A and B. (Fig. 48.)

When the first is subtracted from the second, if the

remainder is +, the second point is higher than the first; if the remainder is — the second point is lower than the first.

In the same manner we may determine the difference in level between the second point and a third point, between the third and a fourth point, and so on, as far as may be desirable.

The total difference of level between the first point and the last is then equal to the algebraic sum of all the partial differences of level.

Example.—A levelling instrument was placed at a station C, mid-way between two bench marks, A and B. (Fig. 48).

Staff-reading at A . 8·25
Ditto at B . 5·10

The instrument was then shifted to a station D, near to B, and 2,000 feet from A.

Staff-reading at B . 4·80

Calculate—

(1.) True difference of level of A and B.

(2.) Error from curvature and refraction of reading at A as seen from D.

(3.) Proper staff-reading at A (including curvature and refraction).

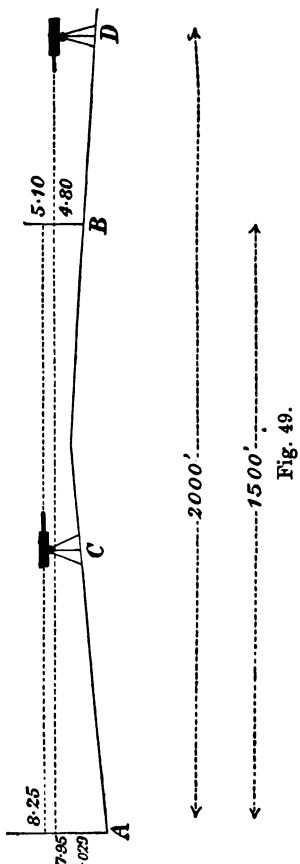


Fig. 49.

(1.) The instrument being mid-way between A and B, the errors from curvature and refraction are neutralised, and the true difference of level of the two bench marks is the difference of their readings on the staff.

$$= 8{\cdot}25 - 5{\cdot}10 = 3{\cdot}15 \text{ feet.}$$

(2.) The error from curvature and refraction of reading at A as seen from D—

$$= \frac{5}{6} \times \frac{2000^2}{41{,}778{,}000} = \frac{5}{6} \times \frac{4{,}000{,}000}{41{,}778{,}000}$$

$$= \frac{5}{6} \times {\cdot}09574 = 0{\cdot}07978 \text{ feet.}$$

(3.) Proper staff-reading at A as seen from D, including effects of curvature and refraction.

$$= (4{\cdot}80 + 3{\cdot}15) + {\cdot}07978.$$

$$= 7{\cdot}95 + {\cdot}07978 = 8{\cdot}02978 \text{ feet.}$$

Note Regarding Adjustment of Level.

Let the readings from C be 8·25 at A and 5·10 at B. The difference of level is 3·15.

If the level is shifted to D, 2,000 feet from A and 500 feet from B, and reads 4·80 at B, then it would read 7·95 at A, if there was no error from curvature and refraction; but the error in 2,000 feet = ·079 ∴ the real reading at A from D should be 8·029. Now if the reading at A from D does not = 8·029, the line of collimation is out of adjustment. Suppose that what we do read is 7·53, or ·50 less than it should read, then ·50 represents the error in A B for 4·80 + 3·15 + ·079 = 8·029 = 7·53 + ·5.

The total error of adjustment in A D

$$= \text{error in A B} \times \left(\frac{\text{A D}}{\text{A B}} = \frac{2000}{1500} = \frac{4}{3}\right) = {\cdot}5 \times \frac{4}{3} = {\cdot}67.$$

Hence 7·53 + ·67 = 8·20 = the true reading at A; and 8·20 — 3·15 = 5·05 = true reading at B from D. Hence 5·05 — 4·80 = ·25 = error in B D.

The curvature and refraction in B D supposed to be inappreciable.

Field Notes.—Gillespie recommends the beginner to sketch the heights and distances measured in a profile or side view, as in Fig. 50. But when the observa-

tions are numerous they should be placed in one of the following tabular forms. These all refer to Fig. 50, and may be compared with that sketch.

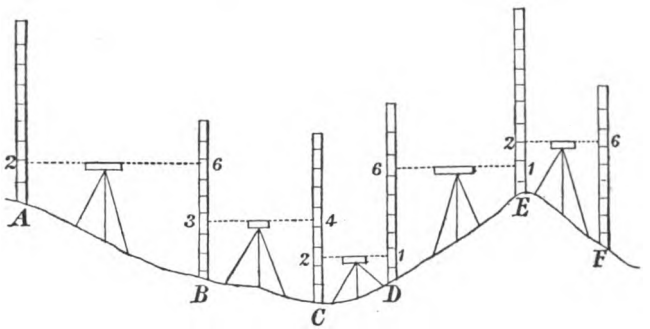


Fig. 50.

First form of Field Book.—In this the names of the points or "stations" whose heights are demanded are placed in the first column, and their height is finally

Stations.	Distances.	Back sights.	Fore sights.	Rise.	Fall.	Reduced Levels.	Remarks.
A						0·00	B.M., mark cut on wall of shepherd's cottage.
B	100	2·00	6·00		—4·00	—4·00	
C	60	3·00	4·00		—1·00	—5·00	
D	40	2·00	1·00	+1·00		—4·00	
E	70	6·00	1·00	+5·00		+1·00	
F	50	2·00	6·00		—4·00	—3·00	B. M., lower hook of gate of farmyard.
		15·00	18·00		—9·00		
					+6·00		
					—3·00		

ascertained in reference to the seventh column. The heights above the starting point are marked +, and those below it are marked —. The back sight to any

station is placed on the line below the point to which it refers. When a back sight exceeds a fore sight, their difference is placed in the column of "rise"; when it is less their difference is a "fall."

The above table shows that B is 4 ft. below A, that C is 5 ft. below A, that E is 1 ft. above A, and so on. To test the calculation, add up the back sights and fore sight. The difference of the same should equal the last total height or reduced level.

Second form of Field Book.—This is presented below. It refers to the same stations and levels noted in the first table, and shown in Fig. 50.

Stations.	Distances.	Back sights.	Height of Inst. above Datum.	Fore sights.	Reduced Levels.	Remarks.
A					0·00	B.M.
B	100	2·00	+ 2·00	6·00	— 4·00	
C	60	3·00	— 1·00	4·00	— 5·00	
D	40	2·00	— 3·00	1·00	— 4·00	
E	70	6·00	+ 2·00	1·00	+ 1·00	
F	50	2·00	+ 3·00	6·00	— 3·00	B.M.
		15·00		18·00	— 3·00	

In the above table it will be seen that a new column is introduced, containing the heights of the instrument above the datum or starting-point. The former columns of "rise" and "fall" are omitted. The above notes are taken thus: The height of the starting-point, or "datum," at A is 0·00. The instrument being set up and levelled, the rod is held at A. The back sight upon it is 2·00; therefore the height of the instrument is also 2·00. The rod is next held at B. The fore sight to it is 6·00. That point is therefore 6·00 below the instrument, or $2{\cdot}00 - 6{\cdot}00 = -4{\cdot}00$ below datum. The instrument is now moved, and again set up, and

the back sight to B, being 3·00, the height of instrument 5 — 4·00 + 3·00 = 1·00, and so on, the height of instrument being always obtained by adding the backsight to the height of the peg on which the rod is held, and the height of the next peg being obtained by substracting the fore sight to the rod held on that peg from the height of instrument.

Third form of Field Book.—In this form the defects of the preceding methods are avoided, and it approximates to a sketch of the operations, the fore-sight being

F. S. −	Distances.	Stations.	Height of Staff.	B. S. +	Height of Inst.	Remarks.
		A	0·00	2·00	+ 2·00	B. M.
6·00	100	B	− 4·00	3·00	− 1·00	
4·00	60	C	− 5·00	2·00	− 3·00	
1·00	40	D	− 4·00	6·00	+ 2·00	
1·00	70	E	+ 1·00	2·00	+ 3·00	
6·00	50	F	− 3·00			B. M.
−18·00				+15·00		
				−18·00		
				− 3·00		

placed before the stations to which they are taken, and the back sights after them. The distances are placed before the stations to which they are taken, or after those from which they are taken. Another advantage is that the stations, their heights, and the distances, are brought together, which facilitates the making of a profile.

In checking the level book after taking levels, the difference between the sums of back and fore sights should equal the difference of the sums of the rises and falls, and each of those quantities should be equal to the difference between the first and last numbers in the columns of reduced levels or height of staff.

Check Levels.—No single set of levels is to be trusted, but they must be tested by another set, run between the bench marks, though not necessarily over the same ground. A set of levels will verify themselves if they come around to the starting-point again. Check levels should be taken of bench marks, of lines of communication, of summits, and of hollows, and for working sections every level should be checked.

Trial Levels.—Their object is to get a general approximate idea of the comparative heights of a portion of the country, as a guide in choosing lines to be levelled more accurately, or for ascertaining the elevation of detached points of primary importance as regards the work in hand. More rapidity and less precision is required in these trial or flying levels.

Levelling Location.—It is the converse of the general problem of levelling, which is to find the difference of heights of two given points. *This* consists in determining the place of a point of any *required* height above or below any given point.

To do this hold the rod on some point of known height above the datum level; sight to it, and then determine the height of the cross hairs. Subtract from this the desired height of the required point, and set the target at the difference. Hold the rod at the place where the height is desired, and raise or lower it till the cross-hair bisects the target. Then the bottom of the rod is at the desired height. Usually a peg is driven till its top is at the given height above the datum.

To Locate a Level Line.—This consists in determining in the ground a series of points which are at the same level, *i.e.* at the same height above some datum. Set one peg at the desired height, as directed above. Sight to the rod held thereon, and make fast the target when

bisected. Then send on the rod—the desired direction, and have it moved up or down the slope of the ground until the target is again bisected. This gives a second point; so go on as far as sight can be correctly taken, keeping unchanged the instrument and target. Make the last point sighted to a "turning-point." Carry the instrument beyond it, set up again, take a B. S., and proceed as at first.

The rod should be held and pegs driven at points so near together that the level line between them will be approximately straight.

Contouring. — Contour lines in hilly districts are usually made 100 feet apart; in lower districts the principal lines are in the 6-inch Ordnance map at 50 feet apart with intermediate lines between. Contour

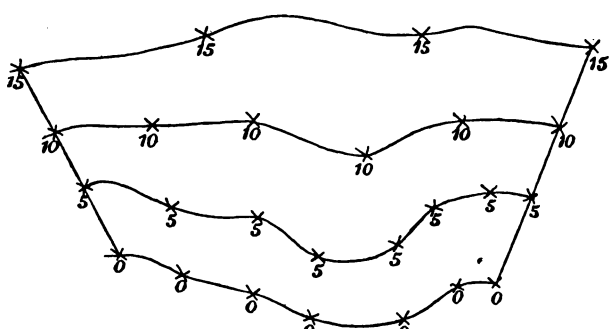


Fig. 51.

lines show the general figure of the surface, and therefore are of little use for the construction of sections if they are at wide intervals. In Fig. 51, the contour lines are 5 yards apart.

Methods for Determining Contour Lines.—They are of two classes:—1. Determining them on the ground at

once; 2. Determining the highest and lowest points, and thence deducting the contour lines.

First Method. General Method. — Determine one point at the desired height of one line, and then "locate" a line at that level. The "reflected hand level," or "reflecting level," or "water level," are sufficiently accurate between "bench mark," not very distant.

One such line having been determined, a point in the next higher or lower one is fixed, and the preceding operations repeated.

On a long narrow strip of ground, such as that required for locating a road, run a section across it at every quarter or half mile, about in the line of greatest slope. Set stakes on these sections at the height of the desired contour lines, and then set intermediate points at these heights between the stakes. These sections *check* the levels. *On a broad surface,* level around it, setting stakes at points of the desired height, and then run sections across it and from them obtain the contour lines as before.

The external here serve as checks to the cross lines.

The contour lines may be surveyed by any method.

Contouring with Plane Table.—It is used to map the points as soon as obtained.

Second Method. General Nature.—This method consists in determining the height and position of the principal points, where the surface of the ground changes its slope in degree or in direction—*i.e.* determining all the highest and lowest points and lines, the tops of the hills and bottoms of hollows, ridges and valleys, &c., and then, by proportion or interpolation, obtaining the places of the points which are at the same desired level. The heights of the principal points are

found by common levelling, and their places fixed by their heights being written upon them.

The first method is more *accurate*; the second is more *rapid.*

Cross Levels.—These show the heights of the ground on a line at right angles to the main line. They give "cross-sections" of it. They may be taken at the same time as the other levels, or independently. In taking "cross levels" where the slopes are steep, as in mountain districts, frequent settings of the instrument are necessary, unless "cross-section rods" are used.

Levelling for Sections. — The object of this is to measure all the ascents and descents of the line, and the distances between the points at which the slope changes, so that a section or profile of it can be made from the observations taken.

A section is a continuous line of levels in which distances as well as heights are measured.

The three principal parts of a section are (1) the datum line; (2) the natural surface of the ground; (3) the line of proposed work. The datum point must be near the terminus of the line of works, and not near the middle of the line.

In *drawings of sections* the *vertical scale* should be vertical on the paper, owing to the irregular expansion and contraction in different directions. The scale of *distances* is divided into miles and furlongs, and these when necessary into chains and tenths of a chain, &c.

The vertical measures in the section must be written in figures; the gradients must also be written, and the changes of gradient marked; as also the greatest depth of cuttings and the greatest height of embankments.

The quantities of earthwork should be calculated from the field-book, and not measured on the paper.

Working Sections.—A working section is a definite profile of the ground, and the levels taken to form this section are termed permanent levels. The heights are taken at every chain from the top of stumps that have previously been driven in, and every minutiæ is particularly defined, such as the tops of banks, depths of ditches, watercourses, roads, pits, &c. A working section should state in writing the level of the ground, and the proposed work, and height of embankment or depth of cutting, at every point whose level has been taken. Those quantities should be found by calculation, not by measurement on paper. The positions and levels of bench marks should also be stated. For working sections every level should be checked.

Trigonometrical Levelling.—This consists by means of a theodolite or transit instrument in measuring the vertical angle between the zenith of the station occupied and the distant object the height of which is to be determined. The horizontal distance to this object must be known, and the measured angle must be increased on account of *refraction*, which may be taken roughly as proportioned to the length of arc of junction, and ordinarily equal to about $\frac{1}{14}$ of the corresponding angle at the earth's centre. We may either measure double the zenith distance—one half of the operation with position of theodolite, say circle left, the other half with circle right (the instrument having been turned 180° in azimuth)—or, if the zenith point (or horizontal point) of the vertical circle be previously determined, it will suffice to measure the single zenith distance (or altitude, depression being a negative altitude).

Irrespective of other *adjustments for the theodolite*,

those for *collimation*, for *verticality of the vertical axis*, and for *horizontality of the horizontal axis* of the telescope must be carefully attended to. The observer should also examine the verticality of the plane of his circle to the last-named axis.

To Measure Horizontal Angles.—Set the transit so that its centre shall be precisely over the angular point. This is done by means of a plumb-line suspended from the centre of the instrument. Level the instrument carefully, sight to a rod, held at some point on one of the lines, as at B in the figure (A being the place of the

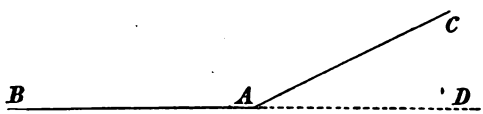


Fig. 52.

transit), and note the reading. Then loosen the clamp of the vernier plate, keeping the other plate clamped; sight to a rod held at some point on the second line, as at C, and again note the reading. The difference of the two readings will give the angle B A C. This is the *angle of intersection.*

To measure the *angle of deflection*, D A C, *i.e.* the angle between A C and B A prolonged. After sighting to B, turn over the telescope. It will now point towards D, in the line B A prolonged. Note the reading; sight to C, and again note the reading. The difference of the readings will give the required angle.

Vertical angles are measured similarly to horizontal ones, only using the vertical instead of the horizontal circle.

Traversing.—In this method of surveying and recording a line, the direction of each successive portion is determined, not by the angle which it makes with

the line preceding it, but with the first line observed, or some other constant line. The operation consists essentially in taking such back sight by the lower motion (which turns the circle without changing the reading), and taking each forward sight by the upper motion, which moves the vernier over the arc measuring the new angle, and thus adds it to or subtracts it from the previous reading.

Set up the instrument at some station, as B; put the vernier at zero, and, by the lower motion, sight back to A. Tighten the lower clamp, reverse the telescope, loosen the upper clamp, sight to C by the upper motion, and clamp the vernier-plate again. Remove

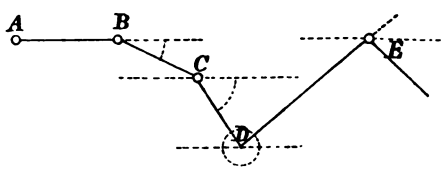


Fig. 53.

the instrument to C, sight back to B by the lower motion. Then clamp below, reverse the telescope, loosen the upper clamp, and sight to D by the upper motion. Then go to D, and proceed as at C; and so on. The reading gives the angles measured to the right or "with the sun," as shown by the arcs in the figure.

Barometric Levelling.—We need here only refer to the barometer as an instrument for measuring heights. In the form of a *mercurial barometer* it may be regarded as essentially a balance in which, under the influence of gravity, the mass of the superincumbent atmosphere is equilibriated by a mass of mercury. In the *aneroid barometer*, on the contrary, the atmospheric pressure is

counterbalanced by the elasticity of a corrugated metallic vessel (generally filled with gas, sometimes supplied with a spring). A change of gravity could not, therefore, be indicated by an instrument of the first form, but would be by one of the second form. It is an instrument of great simplicity and portability, and depends on the known relation between the variations in the atmospheric pressure and the corresponding changes in the boiling-point of water. The results, however, are subject to considerable uncertainty.

CHAPTER IX.

UNITED STATES PUBLIC LANDS SURVEYS.

THE extensive territories of the United States are surveyed upon a peculiar system, planned with reference to the division of the land into squares of uniform size, so arranged that any tract of 160 acres may have its distinct designation and be readily found upon the map or recognised upon the ground by the marks left by surveyors.

These squares are bounded on the east and west sides by lines which are true meridians of longitude radiating from the north pole, and on the north and south sides by lines which are chords of the circular parallels of latitude intersecting such meridians.

In each land district a *principal meridian* line is run, extending through the entire district, and from this meridian, at points 24 miles apart, east and west, *base lines* are run, which also extend through the district. These lines are determined astronomically, and when located serve as axes to which the subdivisions of the district are referred.

Parallel to the axes, and on each side of them, other lines are run 6 miles apart, dividing the whole territory into squares, called *townships*, each containing 36 square miles, or 36 sections.

The meridians are drawn from the base lines north

and south to the depth of two townships; but owing to their not being parallel, they do not meet—that is to say, the meridian drawn north from the first base line to the depth of two townships would not meet the meridian line drawn south from the second base, thus creating corners or offsets between the townships and section outlines, and making necessary a *correction line* at every distance of four townships apart, as shown on the index map.

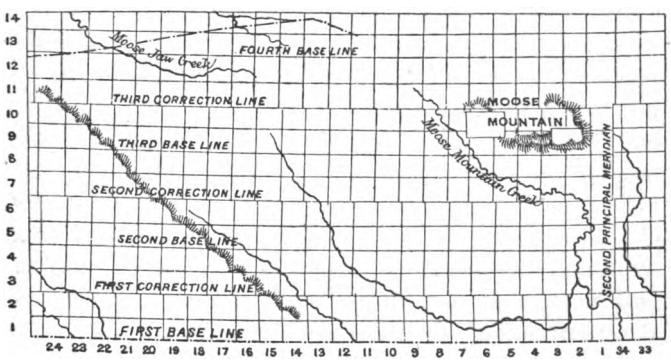

FOURTH BASE LINE
Moose Jaw Creek
THIRD CORRECTION LINE
THIRD BASE LINE
SECOND CORRECTION LINE
SECOND BASE LINE
FIRST CORRECTION LINE
FIRST BASE LINE
MOOSE
MOUNTAIN
Moose Mountain Creek
SECOND PRINCIPAL MERIDIAN
14 13 12 11 10 9 8 7 6 5 4 3 2 1
24 23 22 21 20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 34 33

Fig. 54.

Townships.—A township measures, from centre to centre of the road allowances which form its actual boundary, 483 chains square (= 10,604 yards square), more or less, subject to the deficiency resulting from the convergence or divergence of the meridians, as the case may be, caused by the curvature of the surface of the globe.

The townships lying between two consecutive meridians 6 miles apart constitute a "range," and the ranges are numbered from the principal meridians, both east and west. In each range the townships are

numbered both north and south, from the principal east and west line.

Thus if a township lies 12 miles east of the principal meridian, and 18 miles north of the principal east and west line, it is called township 3 N, range 2 E.

Each township is divided by meridians and east and west lines into squares having a mile on each side. These are called *sections*, and each contains 640 acres, more or less.

Sections.—Each township is divided into 36 "sections" of 640 acres (1 square mile) more or less, the exact area being, like that of the township itself, subject to the convergence or divergence of meridians—together with certain road allowances, having a width of one chain, on each section line running north and south, and on every other section line running east and west.

The sections are laid out the precise width of 80 chains, more or less, on the *base lines* running east and west, and the meridians bounding sections are drawn, both north and south, to the depth of two townships, to the "correction" lines already referred to.

All sections south of a base line will accordingly have their northern boundary lines rather more than 80 chains, while the north and south boundaries of sections in the townships laid off north of the same base line will correspondingly measure less than the normal dimensions of 80 chains. The difference, however, is practically inappreciable, as there is only about half a foot discrepancy between the northern and southern boundary of a quarter section—*i.e.* half a foot in a distance of half a mile.

The sections of a township are numbered from the S.E. corner, running along the southern tier of sections to No. 6, thence backward to No. 12, which lies exactly

north of No. 1, and so on alternately, running from left to right and from right to left, to the north-easterly corner, which is No. 36.

The following diagram shows a township, as surveyed, with road allowance, and the manner in which the sections are numbered.

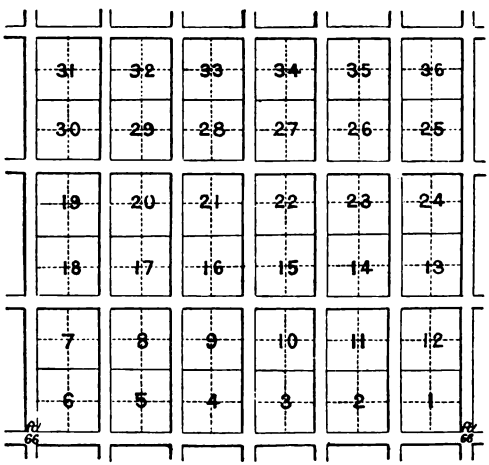
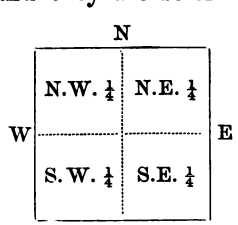


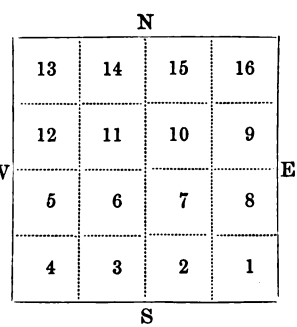
Fig. 55.

Legal Subdivisions of Sections.—Each section of a township is subdivided into "quarter sections," containing 160 acres each, or half a mile square, as shown in the diagram, and they are referred to respectively



as the N.E.$\frac{1}{4}$, the S.E.$\frac{1}{4}$, the S.W.$\frac{1}{4}$, and the N.W.$\frac{1}{4}$ of the section of which they form part.

The terms "half-quarter section," and "quarter-quarter section" are legal designations expressing the quantity of 80 or 40 acres respectively. In the latter case the quarter sections comprising every separate section are, in accordance with the boundary of the same as planted in the original survey, supposed to be further subdivided into four quarter-quarter sections, of 40 acres each, as shown in the following diagram :—



Posts and Mounds.—Surveyed lines on the prairie are marked by posts with mounds of earth built around them, as shown in Fig. 56.

Except in the case of correction lines, section posts and mounds are so placed that lines connecting the cardinal points of the compass will pass through their angles.

On correction lines they are placed square with the line.

In a timbered country the mounds are dispensed with, and the lines marked by blazing the trees on the side next the line and the direction in which it is run, the corners being established by wooden posts, the position of which are defined by bearing trees.

Only a single row of posts to indicate the corners of the townships or sections (except in correction lines)

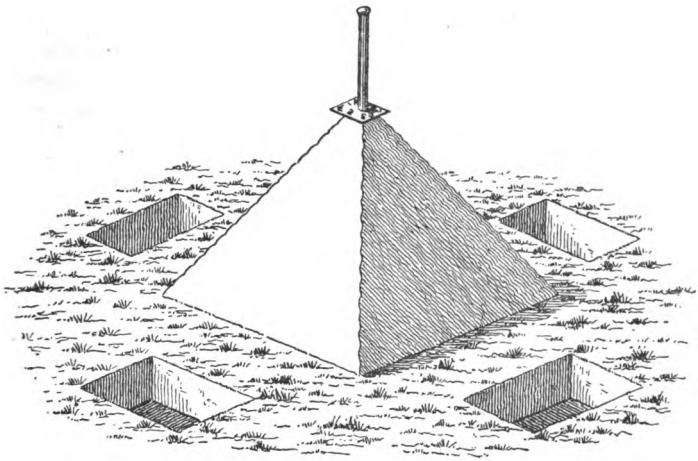
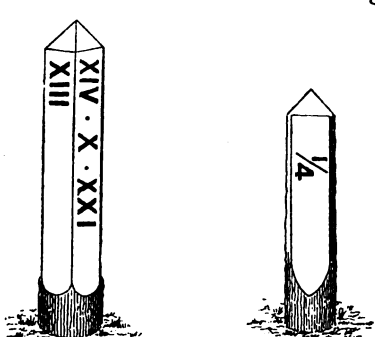


Fig. 56.

is placed on any surveyed line. These posts are placed in the west limit of road allowances leading north and



Figs. 57 and 58.

south, and in the south limit of roads leading east and west.

On correction lines posts are planted on each side of the road allowance, and marked independently for the township on either side.

It frequently occurs that a section corner falls into a lake, slough, or stream. In this case the surveyor builds a circular witness mound on the shore at

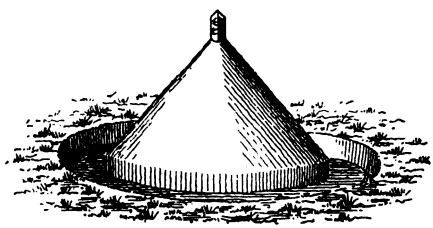


Fig. 59.

the point nearest to the true position of the section corner, the post standing in the mound being marked W.M., the bearing and distance to the site of the true corner being written thereon.

Post Markings.—Wooden posts are marked with Roman numerals cut into their faces.

Where iron posts are used, the figures are punched on a square plate of tin, which rests on the top of the mound, the post passing through its centre. In addition to the section numbers, the plates are marked with the letters N S E and W, and it is necessary in ascertaining the number of a section to see that the plate is turned so that these letters correspond with the cardinal points which they are intended to indicate.

Quarter section corners are designated by wooden posts, flattened on two sides. They are marked with the fraction $\frac{1}{4}$, and stand with their flattened sides facing the direction in which the section line is run.

The position in which the mounds and posts stand

Township 7 Range 15 W. 2nd Mer.

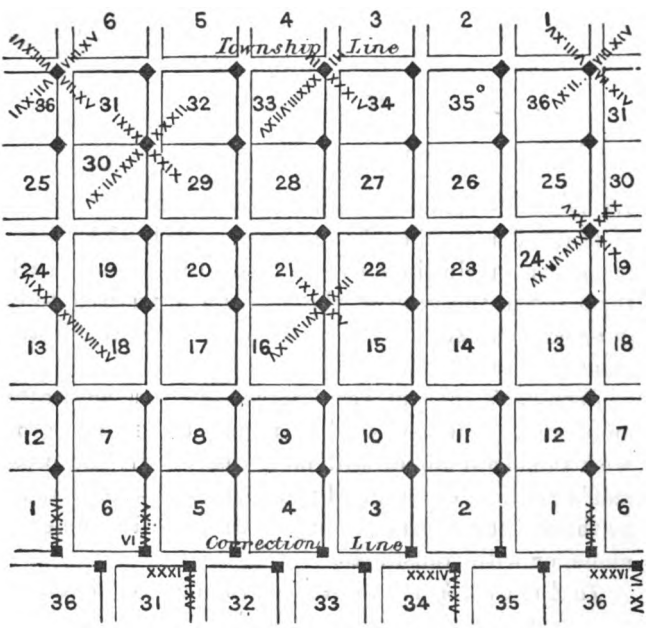


Diagram shewing manner in which posts are marked

Fig. 60.

with reference to the section lines, and the manner in which they are marked, is shown in the above diagram.

CHAPTER X.

LAYING OUT LAND.

Its Nature.—This operation is precisely the reverse of that of surveying, properly so called. The latter measures certain lines as they are; the former marks them out in the ground where they are required to be, in order to satisfy certain conditions. The same instruments, however, are used as in surveying.

Perpendiculars and *parallels* are the lines most often employed. The perpendiculars may be set out either with the chain alone, still more easily with the cross-staff or optical square, and most precisely with a transit or theodolite. Parallels may be set out with the chain alone, or with transit, &c.

To Lay Out Squares.—Reduce the desired content to square chains, and extract its square root. This will be the length of the required side which is to be set out by one of the methods indicated above.

An *acre* laid out in the form of a square is frequently desired by farmers. Its side must be made $316\frac{1}{4}$ links of a Gunter's chain, or $208\frac{71}{100}$ ft., or $69\frac{37}{100}$ yds. It is often taken at 70 paces.

The number of plants, loads of manure, &c., which an acre will contain at any uniform distance apart can be at once found by dividing 209 by this distance in feet, and multiplying the quotient by itself, or by

dividing 43,560 by the square of the distance in feet. Thus, at 3 feet apart, an acre would contain 4,840 plants, &c.; at 10 feet apart, 436; at a rod apart, 160, and so on. If the distance apart be unequal, divide 43,560 by the product of these distances in feet; thus, if the plants were in rows 6 feet apart, and the plants in the rows were 3 feet apart, 2,420 of them would grow on one acre.

To Lay Out Rectangles.—The content length being given, both are measured by the same unit, divide the former by the latter, and the quotient will be the required breadth. Thus, 1 acre or 10 square chains, if 5 chains long, must be 2 chains wide.

The content being given, and the length to be a certain number of times the breadth. Divide the content in square chains, &c., by the ratio of the length to the breadth, and the square root of the quotient will be the shorter side desired, whence the larger side is also known. Thus, let it be required to lay out 30 acres in the form of a rectangle three times as long as broad. 30 acres = 300 square chains. The desired rectangle will contain 3 square each of 100 square chains, having sides of 10 chains. The rectangle will therefore be 10 chains wide and 30 long.

An *acre* laid out in a rectangle twice as long as broad will be 224 links by 448 links nearly, or 147½ ft. by 295 ft., or 49⅓ yds. by 98⅔ yds. 50 paces by 100 is often used as an approximation easy to be remembered.

To Lay Out Triangles.—The content and the base being given, divide the former by half the latter to get the height. At any point of the base erect a perpendicular of the length thus obtained, and it will be the vertex of the required triangle.

The content being given, and the base having to be *m* times the height, the height will equal the square root of the quotient obtained by dividing twice the given area by *m*.

The content being given and the triangle to be equilateral, take the square root of the content and multiply it by 1·520. The product will be the length of the side required. This rule makes the sides of an equilateral triangle containing *one acre* to be 480$\frac{1}{2}$ links. A quarter of an acre laid out in the same form would have each side 240 links long.

The content and base being given, and one side having to make a given angle, as B, with the base A B, the length of the side B C $= \frac{2 \times \text{A B C}}{\text{A B sin. B}}$

Example.—Eighty acres are to be laid out in the form of a triangle, on a base, A B, of 60 chains, bearing N. 80° W., the bearing of the side B C being N. 70° E. Here the angle B is found from the bearings to be 30°. Hence B C = 53·33. The figure is on a scale of 50 chains to 1 inch, = 1 : 39,600.

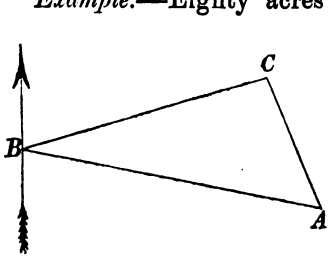


Fig. 61.

Any right-line figure may be laid out by analogous methods.

To Lay Out Circles.—Multiply the given content by 7, divide the product by 22, and take the square root of the quotient. This will give the radius with which the circle can be described on the ground with a rope or chain.

A circle containing *one acre* has a radius of 178$\frac{1}{4}$ links.

A circle containing a quarter of an acre will have a radius of 89 links.

There is much truth in the proverbial advantage of a farm or estate lying within a ring fence. The inconvenience which arises from fields of one farm running into or lying in the middle of another farm is often very great, and it is still too common in England. There can be no excuse for it, certainly, where the farms belong to the same owner. Both that and irregularities in the boundaries should be rectified at the very first opportunity; and, if they are on different proprietors' lands, such glaring inconveniences should be made the subject of an excaulm, or exchange, in the interest of the owners as much as of the farmers.

As to the size of farms, both large and small ones have their fitness and utility; but grazing farms should be relatively larger than those devoted to tillage.

In the interior arrangements of the farm lands, a great consideration is to have the fields properly set off, not merely as regards the division of arable and pasture land, but so as to mark the different soils that occur in each. It makes all the difference sometimes in the working of the fields, and in the equally good yield of the crops if the divisions have been made as far as possible in conformity with the surface soils.

The old style of laying out farms in England—if style it can be called—cannot certainly claim much on the score of utility; and it is a true type of the falsely named picturesque, with its miry lanes, its wide straggling hedgerows and ditches, the land cut up into numberless misshapen fields, and here and there waste plots of ground that seem to have been left out of the general plan because they would not fit in anywhere.

There can be little doubt that enclosures are bene-

ficial to a certain extent—in some districts more so than in others; but whatever the size and number of the enclosures, they should be laid out so as to utilize the whole of the ground that makes up the farm and is free for this purpose. And if the wasteful agriculture which is so strongly encouraged by an ill arrangement of the fields is to be fully avoided, the same principle must regulate the laying out of roads and buildings.

It must be confessed that there is little plan or precision in our field system. If the limits of the field are defined, and perhaps the boundary fenced, it is thought to be quite enough for the purposes of agriculture; and if the shape of the field is at all nearly square or rectangular, it is considered a very well laid-out piece of land.

But while it is proper that the size, shape, and boundary of the enclosure should be attended to, this does not supply the place of an interior arrangement of the fields. Only those who have tried the plan of marking out the whole field-surface in squares or divisions of fractions of an acre know its advantages, in point of accuracy and readiness in conducting almost every branch of field-work, and in making trials of seeds, manures, &c. We shall notice a few of the advantages arising from such a plan.

In the first place, it enables the farmer to see at a glance what quantity of field-work is done in a given time by the hands or teams employed, no matter what description of work they are at—hoeing, reaping, ploughing, manuring, or anything else; and it is, at the same time, a great help to a better inspection of the work done. It also facilitates and encourages the use of piece-work in the employment of field hands. A square can be taken up, worked off, inspected, and paid

for at the rate agreed on per acre with a simplicity that compares favourably with the slipshod fashion of day hands at work without a gauge, or piece-workers whose task is an uncertainty until the ground is measured.

It likewise affords a ready facility for making experiments on a small scale with seeds and manures, for trying different modes of culture, and for ascertaining the weight of produce obtained from the land, or the weight of crop consumed in the feeding of stock. These are only a few of the advantages. The plan, when adopted, will be found convenient for many purposes besides those enumerated.

But the question is, How to lay out fields in small squares so as to provide these advantages. With the use of improved machinery it does not do to have ditches, nor even deep furrows, as the bounds of these squares; and otherwise, of course cropped land has to be marked out annually, or at least as often as the field is resown. It may be done, however, not by furrows, but simply by lining out the ground after the crop is sown, and then drilling in the lines, with a small hand single drill, a single row of the same seed as the crop may be—grass, corn, turnips, &c. The same variety of seeds will be quite visible in the lines, especially where otherwise throughout the field the seed-corn or grass was broadcasted; and in the case of root crops, it may be nicely arranged by drilling, say in a field of yellow turnips, swede-seed in the marked lines, &c.

The expense and trouble of marking out lines is very little, and is soon amply repaid. Let a man, or a man and a boy, line out the field as soon as it is sown, in *tenths* or *poles*, &c., as decided on. Then take the hand drill and run along the lines, first the length of the

field, then across, drilling somewhat thicker in the lines than it was sown over the field. The lines must be kept perfectly straight; but if the ground is properly lined out first, that is easy to do. The rate at which a man and a boy will line and drill-mark a field will of course depend on its total acreage, and on the size of the divisions into which it has to be lined or marked.

Of course if a more permanent system of marking without loss of land could be devised it would be better; but until that is discovered no labour on the farm will be better bestowed than that given to forming the line divisions here indicated.

CHAPTER XI.

ESTIMATING WEIGHT, QUANTITY, AND VALUES.

A KNOWLEDGE of measuring solids is indispensable to the land agent, surveyor, and farmer, as the dimensions and weight of all materials and objects are calculated by it.

Mensuration of solids comprehends the measure by length, breadth, and thickness of all bodies, whether solid, liquid, or gaseous. The general rule is, to "find the area of one end, and multiply that by the length." This rule is of universal application, whether to earthwork, ricks of corn or hay, heaps of dung, of stone, or of burnt clay, and to timber, &c. The area of one end, or of one surface, whether the end, side, top, or bottom, is found on exactly the same principle as in ascertaining the superficial contents of land; and if the figure diminishes in the course of its length, as the top of a rick, or the trunk of a tree, the mean length or half is taken as a multiplier.

Weight of Cattle.—The rule for ascertaining the weight of an animal by measurement is to multiply the square of the girth by the length, and this product by the decimal ·238, which will give the weight of the four quarters in imperial stones.

To ascertain the dead-weight by weighing—multiply the live-weight by the decimal ·605, and the product

will give the dead-weight of the four quarters in imperial stones.

It is simpler to ascertain the weight by measurement.

We here explain the calculation of the dimensions of an animal and the principle upon which it is founded, by which anyone may find the weight without the assistance of tables. The length and girth of the animal being measured—the first from the shoulder-top to the tail-head, and the second immediately behind the shoulder—these dimensions bring the figure of the animal into the form of a cylinder, or nearly so. The rule for finding the contents of a cylinder is to find the area of the end and to multiply that sum by the length. The common method is, to multiply the square of the diameter by ·7854 (the area of a circle whose diameter is unity), and this product by the length for the solid content. But in measuring cattle the girth or circumference, and not the diameter, is obtained; and as the rule for finding the diameter correctly from the circumference involves itself into long decimal multipliers, the process, especially when feet and odd inches are the dimensions, is complicated and tedious. The more simple method, therefore, is to multiply the square of the circumference by ·0795775,* and that product multiplied by the length gives the contents; which again multiplied by the established weight of a cubic foot or other measure will give the weight of the animal.

To find the proportional weight of a cubic foot, &c.: —Find, by the above rule, the number of cubic feet

* This number is obtained in the following way: The area of a circle equals the square of the circumference divided by four times 3·1416 (the circumference of a circle whose diameter is unity); or 1 divided by 12·5664 (=4×3·1416)=·0795775. Hence the square of the circumference multiplied by ·0795775 gives the area of the circle.

which the animal contains, and weigh the four quarters after it is killed, and the former divided by the latter would give the weight per cubic foot. Thus, if an ox measures 8 feet girth, 6 feet length: $8 \times 8 = 64 \times \cdot0795775 = 5\cdot09296 \times 6 = 30\cdot55776$ cubic feet in the animal;* and if the four quarters of the killed animal weighed 91 stones 6½ lbs., this weight divided by the number of cubic feet in the animal gives the weight of a cubic foot. Hence 91 stones 6½ lbs. = $91\cdot4642857 \div 30\cdot5576$ give 2·993 stones per cubic foot. And this is the actual weight assumed for a cubic foot.

The calculation may be shortened. As ·0795775 and 2·993 are both constant multipliers in the operation, they may be multiplied together and the product used in one multiplier, thus: $\cdot0795775 \times 2\cdot993 = \cdot2381754675$. But ·238, or three figures only, may be near enough for a multiplier. Thus $8 \times 8 = 64 \times 6 = 384 \times \cdot238 = 91\cdot392$, or 91 stones 5½ lbs. (In place of ·238 some use ·24, which gives a higher weight.) Thus, then, to find the weight of a fat animal, multiply the square of the girth by the length, and that product by ·238, or take $\frac{238}{1000}$th part of it, or use any lower and more convenient denomination of the same value.

Another rule is to multiply the square of the girth by five times the length, and divide the product by 21, to get the weight of the four quarters; *i.e.* multiply the square of the girth by the length and take the $\frac{5}{21}$st part of the product for the weight. Now $\frac{5}{21}$ converted into decimals is ·23809523, which exactly agrees, in

* The calculation may be performed by duodecimals, or by reducing odd inches, if any, to decimals or fractions; or both length and girth may be reduced to inches, and then as above. To bring cubic inches to feet, divide the product by 1728.

as far as the decimal numbers necessary for the calculation are required, with the numbers we have given.

Weight of Hay-ricks. — Various modes may be adopted for determining this, but the only accurate one is by the use of the platform scales. The number of tons may be nearly determined by ascertaining the number of cubic feet or yards in the rick, and obtaining the weight per foot by actual weighing if necessary.

Weight per foot.		Yards to a ton.	Weight per foot.		Yards to a ton.
lbs.	oz.		lbs.	oz.	
5	3	= 16	7	8	= 11
5	8½	= 15	8	4	= 10
6	0	= 14	9	3	= 9
6	6	= 13	10	5	= 8
6	14	= 12			

The number of yards per ton will depend on the solidity of settlement of the stack. If a good-sized stack has well settled, about 12 cubic yards to a ton will be fair.

The following rule will give the weight approximately by measurement.

With the tape measure the length and breadth, then the height to the eaves, and from the eaves to the top.

To calculate the quantity proceed thus :—

To the height from the ground to the eaves, add one-third of the height from the eaves to the top : multiply this sum by the breadth, and that product by the length. This will give the area in feet, which, divided by 27 (cubic feet in a yard), the quotient will be in yards. Divide this by 10 to bring it into tons.

1. *Example.*—Suppose a stack of hay 30 ft. in length, 20 ft. in breadth, the height to the eaves 14 ft., height from the eaves to the top 9 ft. Required the quantity in tons.

14 ft. height to the eaves
3 add ⅓ of height to roof

17
20 breadth

340
30 length

27) 102,00 (377·7 ÷ 10 = Tons 37.7
81

210
189

210
189

21

27

20 cwt.
7

10) 140

14 cwt. = $\frac{7}{10}$ of a ton.

2. *Example.*—Required the quantity of hay in a stack, the dimensions of which are as follows:—

Height to the eaves	12 feet.
From the eaves to the top . .	9 ,,
Length	25 ,,
Breadth	14 ,,

Average compactness 10 yards to a ton. *Answer*, 19 tons 8 cwt.

Thatching.—Thatchers' work is measured by the square of 100 square feet.

1. To find the quantity of thatching on square or oblong ricks:—

Rule.—Multiply the width over the top from eave to eave by the length at the eaves, both in feet, and divide the product by 100 for the quantity in squares of 100 square feet. If the ends of the rick are thatched, add the breadth of the rick to the length for the multiplier.

Example.—Required the amount of thatching on a rick measuring 30 ft. over top from eave to eave, length of side 40 ft., and width at eaves 12 ft. *Answer*, $15\frac{60}{100}$ squares.

2. To find the quantity of thatching when the roof of the rick or stack is conical:—

Rule.—Multiply the circumference of the eaves by half the slant height, both in feet, and divide the product by 100 for the quantity in squares of 100 square feet.

Example.—Required the amount of thatching on a rick of 36 ft. circumference, 12 ft. slant height. *Answer*, $2\frac{1}{1}\frac{6}{0}\frac{}{0}$ squares.

Measurement of Grain in a Bin or Heap.—Multiply the length, breadth, and depth in feet, and that product by 0·8. Suppose the bin is 20 ft. long, 4 ft. wide, and 6 ft. deep; this will give, when multiplied together, 480 cubic ft. To reduce this amount to bushels multiply by 0·8, which gives 384 in answer. It takes 2,150 cubic inches to make a straked bushel, and a cubic foot has 1,728 cubic inches; hence the bushel is to the foot as 5 to 4, which is the explanation of the use of the fraction 0·8.

Weight is the only true standard of the quality of corn. The heaviest wheat in the smallest compass will always yield the greatest proportion of flour, and the millers are so well aware of this that they always stipulate for a nominal measure to be made up to a certain specified weight; but the heavy corn will always be worth more than the same weight of lighter corn.

To measure Dung-heaps, &c.—Measure the length and breadth, and take three or four depths, according to the inequalities of the surface. The mean of these depths, multiplied by the length and that product by the breadth, will give the cubical content of the heap.

If the area is in feet, divide by 27, and the quotient will be the quantity in loads.

Example.—A dung-heap is 50 ft. long, 25 ft. broad,

and of the different depths of $3\frac{1}{6}$, 4, $4\frac{1}{2}$, and 5 ft. Required the number of cartloads.

Ft.	in.
3	2
4	0
4	6
5	0
4)16	8
4	2 average depth.

Ft.
50 length.
25 breadth.

1250 ft. in.
$4\frac{1}{6} = 4$ 2 aver. depth.

5000
208·3

27)5208·3(192·9 cartloads.
27

250
243

78
54

24·3
24·3

Earthwork.—In calculations of earthwork we require to know three things—

(1.) The base.

(2.) The slope.

(3.) The depth of cutting or height of embankment.

In all rectilineal excavations, such as trenches, &c., or any other regular figure, the common rule will produce accurate quantities, as—

Multiply the length by the thickness and the product by the breadth. If the dimensions are taken in feet, to reduce it to cube yards, divide the product by 27. A cube yard of earth is equivalent to a load.

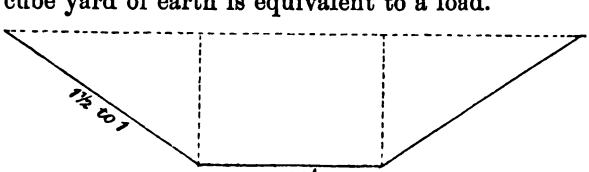


Fig. 62.

Example.—Given base of a cutting 30 ft., slopes $1\frac{1}{2}$ to 1, as in Fig. 62. Natural ground level transversely. Depths at intervals of 100 ft., as follows:—

Distances. (Feet.)	Depths. (Feet.)	Areas. (Sq. Feet.)
0	0	0
100	14	714
200	24	1584
300	30	2250
400	32	2496
500	24	1584
600	0	0

Calculate the sectional area in square feet; also the volumes in three divisions each of 200 feet long in cubic feet to the nearest whole number; also the total volume in cubic yards to the nearest whole number:—

(1.) AREA $= (30 \times h) + (1\frac{1}{2} \times h^2)$.

Area of section 0 = 0
" " 100 = 420 + 294 = 714 sq. ft.
" " 200 = 720 + 864 = 1584 "
" " 300 = 900 + 1350 = 2250 "
" " 400 = 960 + 1536 = 2496 "
" " 500 = 720 + 864 = 1584 "
" " 600 = 0

(2.) VOLUME $= x. \frac{A. + 4A_1 + A_2}{6}$

Volume of first 200 feet of length

$$= 200. \frac{(714 \times 4) + 1584}{6} = 148{,}000 \text{ cub. ft.}$$

Volume of middle length

$$= 200. \frac{1584 + (4 \times 2250) + 2496}{6} = 436{,}000 \text{ cub. ft.}$$

$$= 200 \frac{\text{Volume of third length}}{2496 + 1584 \times 4 \over 4} = 294{,}400 \text{ cub. ft.}$$

(3.) THE TOTAL VOLUME IN CUBIC YARDS.

$$= \frac{148000 + 436000 + 294400}{27}$$

$$= \frac{878400}{27} = 32533{\cdot}32 \text{ cub. yds.}$$

Estimating the quantity of work which ought to be done in a given time by teams of labourers is an art that ought to be familiar to every agriculturist. In general no absolute rule can be laid down, because so much depends on soils, roads, cattle, and other circumstances; but in every particular case the rate or market price of labour per day being given, and the quantity of work ascertained which a man or a horse can fairly perform in a given time, a rate per square yard, pole, or acre, or per solid quantity of materials which are to be moved, can easily be determined on. A farmer should know by memory the number of ridges, or of single furrows or bouts, which it requires to make an acre on every field of his farm. This will aid him in every operation that requires to be performed on these fields, the quantity of manure, seed, ploughings, harrowings, hoeings, mowing, reaping, raking, &c., as well as in estimating the produce, whether corn, hay, roots, or the number of cattle or sheep that may be grazed there for any given time (Louden). It will be seen that the proper laying out of farm lands has a great deal to do with this.

Draining work, ditching and hedging, &c., ought to be subjected to similar calculations, so as, if possible, to let out all work not performed with the master's own

men and cattle, by contract or quantity, instead of by time. As spade work is nearly the same in most parts of the country, certain general rules have been laid down which, though seldom strictly followed up, it may be useful to know. Thus in moving soil, as in digging a drain or the like, if the soil is soft and no other tool than the spade is necessary, a man will throw up a cubic yard of 27 solid feet in an hour, or 10 cubic yards in a day. But if picking be necessary an additional man will be required, and very strong gravel will require two. The rates of a cubic yard, depending thus upon each circumstance, will be in the rates of the arithmetical numbers 1, 2, 3, so that if a labourer earns 3s. 4d. a day the cost per cubic yard will be 4d. in the first case, 8d. in the second case, and 1s. in the third case.

Measurement of Timber.—It is desirable that a regular and recognised method of measuring be adopted, and the simplest and easiest way is to take the whole length of the tree to the height where it girths 26 inches round the bark. Then take the mean girth of the tree between this point and the base. If the measurement is taken outside the bark, one inch should be allowed for every foot of circumference. In practice, I girth the standing tree at 6 feet from the ground, which gives the medium girth of the lower 12 feet; then compute by sight the upper part of the tree, which a little experience soon enables any one to do sufficiently near for all practical purposes.

Before commencing to measure, the forester provides himself with a foot-rule and slide, leather strap marked or graduated, a pole, a marking axe, and a red lead pencil, or a small brush and paint.

The strap may be any convenient length, from 15 to 20 feet, $\frac{3}{4}$ inch broad, and of the strength of a small

bridle-rein, with a piece of lead attached to one end of it. Previous to marking the strap with the necessary figures, the leather should be alternately wetted and dried, otherwise it is apt to shrink and expand when in use, according to the state of the weather. A seasoned gig-rein, when reduced to proper dimensions, forms an excellent strap. Such straps are not generally to be bought in the shops, but must be home-made.

6 being the last figure upon the strap, is exactly 26 inches from the end, including the lead and is the side of the square. The next figure is 7, and so on. The cross stroke indicates half inches, and quarter inches are indicated by the dot.

The pole used for taking the height is 14 feet long, marked *ft.* and *half ft.* The lowest mark is at 6 feet, at which height the trees are mostly girthed. Thus, by an expeditious and simple process the contents of the first 12 feet of the trunk are found. This is a ready method of measuring standing timber either for sale, transfer, or other purposes.

Though the use of the slide-rule is recommended for casting up the contents of a tree, yet in extensive practice it is seldom used. Having the length of the section (or whole tree) as indicated by the pole, and the side of the square as shown by the girth, the relative contents soon become so familiar that no casting up is required.

In making the strap it is advantageous to mark one side with white paint for measuring peeled timber, when no allowance is made for the bark; thus the side of the square of a tree *three feet* (3 feet) in girth is 9 inches. The other side may be marked with red paint, allowing for bark at the rate of 1 inch to the foot in girth.

The true and full content of a round tree can be found very nearly by Dr. Hutton's rule, which is—

Multiply the square of one-fifth the mean girth by twice the length.

Take a tree 44 in. in girth and 32 ft. long,

$$8{\cdot}8^2 \times \frac{64}{144} = 34{\cdot}41 \text{ cubic ft.}$$

But rough timber is never bought and sold by this rule, for it allows nothing for loss and waste in squaring the tree. In order to provide for that, another rule of measurement is adopted in practice, and it is this—

Multiply the square of the quarter girth by the length, and take the product for the volume. Thus the above tree gives by this rule—

$$11^2 \times \frac{32}{144} = 26{\cdot}88 \text{ cubic ft.,}$$

or little more than three-fourths of the full content of the tree. It is often urged against this rule that it allows more than is needed to make good the loss in squaring the tree, but it may be shown that it gives a higher result than the tree can actually be hewn to. It assumes that the quarter-girth and the side of the inscribed square are equal, whereas it is self-evident that the inscribed square is less than the circle. The greatest square a tree 44 inches in circumference can be hewn to is 9·9 inches, instead of 11 inches, as is assumed by the common rule of the quarter-girth, and therefore the actual volume of squared timber in the tree is only

$$9{\cdot}9^2 \times \frac{32}{144} = 21{\cdot}76 \text{ cubic ft.,}$$

or 5·08 cubic ft. less than by the quarter-girth rule. The true squared content will, therefore, be fully 36 per cent. less than the full content of the unhewn tree.

This great difference may very well be more than will cover the actual cost of and loss by squaring, if the slabs and chips are saleable, as they generally are, and hence practice has adopted a rule which, although erroneous in itself, gives something between the two, and for general use is the most fair one as between buyer and seller.

In measuring standing timber there is very little difficulty: use a ladder and pole for the length, and take the girth in three or four places.

A *cord* of wood is 8 ft. long × 4 ft. wide × 4 ft high = 128 cubic ft.

Valuing Plantations.—It can scarcely be said that any forester thoroughly knows his duties till he can correctly value the various crops of wood of different ages that are under his charge. There are various ways of doing this.

On taking charge of the woods and plantations on an estate, it is necessary, in the first place, to become acquainted with every individual plantation, great and small, on the estate, and to ascertain (1) the name by which it is known on the estate plan, and generally or locally as well; (2) the date of planting, and whether planted in spring or autumn, as that makes a difference of one year's growth; (3) the acreage; (4) the proportion of different classes or species of trees in the plantation; (5) the aggregate number of trees; (6) the total value of the trees; (7) the average annual growth each plantation is making at the time of inspection; in lineal feet or cubic contents; (8) the money value per acre of the annual growth of each plantation; (9) the transferable value per acre; (10) the highest prospective value the plantation is ever likely to attain.

To the value of the growth of all plantations of

which the thinnings have not yet paid for the original outlay in forming the cost of thinning is added, but not when the thinnings have paid such outlay.

It may be necessary to explain how the various data are obtained, and especially how the number of trees upon the ground are ascertained.

A book of convenient size for the pocket is provided, and ruled horizontally but not vertically, which can best be done on the spot; and with the assistance of two men a line through the greatest length of the plantation is taken, the first man, or leader, calling out the species of each tree in the line as he comes to it—oak, ash, larch, spruce, &c.—and the second man intimating the distance in feet and inches between each two trees throughout the whole length of the plantation. The leading man also girths the tree as he goes up to it, always at 4 feet from the ground, intimating the figures, and walks on to the next tree. The line pursued is not direct, but zigzag, each tree being taken as arrived at. The valuer in the meantime is entering the various data in his book—such as species, distance, height, and girth—

Species.	Distance.	Height.	Girth.
Sp. (Spruce).	10	16	15
L. (Larch).	18	15	20
Bi. (Birch).	13	14	16

and so on.

The plantation is again traversed in other directions, and when the whole is done the various averages are taken, by adding all the respective measurements, and dividing the same by the number of them.

It is to be observed that the trees are girthed at 4 feet from the ground, at which the height over the

bark is equal to the girth at the ground *minus* the bark. This applies only to such plantations as have been at least once thinned but are below timber size.

In dealing with plantations that have not been thinned at all, the practice is to go carefully through every part, and note the lengths of the last top growths, and the full height of the trees, to the number of about 800 to the acre—the quantity that should remain upon the ground after being thinned the first time.

The method of casting up the value of a young plantation is a matter of simple proportion, stated thus—If a plantation is valued prospectively at sixty years' growth to be worth £60 an acre, what ought it to be worth at forty years' growth?

Plantations below thirty years growth have the original cost of forming added or subtracted as the case requires, and those over thirty years, or such as have been sufficiently thinned, are simply valued according to their present or prospective worth.

No further revenue is expected from the thinnings of a plantation after thirty years old, but an equivalent benefit is derived from it, in the form of grazing for sheep or cattle, which is indeed often of more value than the thinnings.

This mode of valuing young plantations may be objected to on account of no allowance being made for interest on the original outlay, and no sum being set apart for rent. These, however, are included in the *one item*—viz. the value of the annual growth. Say, the annual value of the growth of a plantation is 20s. per acre; against this there is the ground-rent 7s. 6d. per acre, and *interest* on cost of planting, 70s. per acre at 5 per cent., = 3s. 6d., making altogether 11s. per acre

chargeable against the 20s. worth of produce, and leaving 9s. per acre in favour of planting.

There are various ways of valuing mature timber and old plantations, but the only plan to be recommended is to take the quantity of measurable timber tree by tree, and put a price upon it according to the kind and quality of the wood.

The age at which a plantation will come to maturity and at which it should be cut down to yield most profit, is a question that admits of no general answer; for trees vary in growth and early maturity according to soil, situation, climate, &c., so that no fixed period of cutting can be generally applied to any class of trees grown in different places and under more or less varying conditions. Within the range of our own climate, however, the quality and depth of the soil that the tree stands in has more influence on the age of maturity than any other single circumstance. For it has been found that when an oak tree in good strong soil 2½ feet deep will cut most profitably at fifty years, in an equally good and strong soil 3½ feet deep the same tree requires about seventy years to come to maturity; and if the soil is 4½ feet deep, one hundred years; but in lighter and sandy soils of the same depths, the periods of maturing are lessened to forty, sixty, and eighty years. The hard-wood trees are all slow growers, and it is generally held that a hard-wood plantation requires sixty years to come to maturity. A fir plantation will be at its best in about half that time, or thirty years. Where the plantation is a mixed one, the relative quantity of fast and slow-growing trees, and the effect which the greater proportion of either will have on the time of average maturity, must necessarily be considered.

To Calculate the Cost of Buildings by the Square.—The dimensions of the different compartments are taken and the length of each multiplied by the breadth, so as to give their superficial areas; and an amount per square foot, or generally per square of 100 ft.—that is, 10 ft. each way—is assigned to each. But the same difficulty attends this mode of computation (as estimating by comparison), even perhaps to a greater extent, for the sum placed against each square must be guided entirely by what is supposed to be the value of the class of building (farm-house, cottage, steading, &c.) it is intended to erect.

Professor Kerr very properly varies the allowance per square in proportion to the cost, extent, and finish of the house, and begins at £40 per square (and upwards) for family rooms, and £28 for servants' rooms, of a house of about the value of £1,250, increasing to £100 and £50 per respective squares in a house estimated at £40,000.

His plan is to take the dimensions of every room and portion of the house internally, multiplying their relative length by the breadth, then squaring the floor-spaces of passages and stairs in the same manner, and adding to the total $\frac{1}{5}$th of the whole for walls and waste.

TABLE OF THE RELATIVE PROPORTIONS OF THE CIRCLE, ITS EQUAL AND INSCRIBED SQUARES.

1. Diameter of a circle	× ·8862	= the side of equal square.	
2. Circumference	× ·2821		
3. Diameter	× ·7071	= the side of inscribed square.	
4. Circumference	× ·2251		
5. Area	× ·6366		
6. Side of inscribed square	× 1·4142	=	the diameter of a circumscribed circle.
7. Side of inscribed square	× 4·443	=	the circumference of an equal circle.
8. Side of a square	× 1·128	=	the diameter of an equal circle.
9. Side of a square	× 3·545	=	the circumference of an equal circle.

CHAPTER XII.

WEIGHTS AND MEASURES.

Lineal Measure.

Marked.			
inc.	12 inches	=	1 foot.
ft.	3 feet	=	1 yard.
yd.	5½ yards	=	1 pole.
pl.	40 poles	=	1 furlong.
fur.	8 furlongs	=	1 mile, *m.*

4 inches = 1 hand, 6 feet = 1 fathom, 3 miles = 1 league.

Inches.		Foot.		Yard.		Pole.		Furlong.		Mile.
12	=	1								
36	=	3	=	1						
198	=	16½	=	5½	=	1				
7920	=	660	=	220	=	40	=	1		
63360	=	5280	=	1760	=	320	=	8	=	1

The inch is divided by mechanics into halves, quarters, 8ths, and 16ths. It is also divided into 10ths, and into 12ths, called lines.

Land is measured by the Imperial chain of 100 links = 66 feet, and therefore 1 link = 7·92 inches.

In geographical and nautical measurements, 60 minutes = 1 degree = 60 geographical miles. But 360 degrees = the circumference of a circle; therefore the mean circumference of the earth being 24,856 English miles, 1 degree = $69\frac{2}{5}$ English miles, or 60 geographical miles.

Measures of Surface, or Square Measure.

By square measure is meant length and breadth taken together. Thus, 1 foot, or 12 inches long by 12 inches broad = 12 × 12 = 144 square inches = 1 square foot, &c.

Marked.		
sq. inc.	144 square inches	= 1 square foot.
sq. ft.	9 square feet	= 1 square yard.
sq. yds.	30¼ square yards	= 1 square pole.
pls.	40 square poles	= 1 rood.
rds.	4 roods	= 1 acre, *ac.*

Sq. In.	Sq. Ft.	Sq. Yd.	Sq. Pl.	Rood.	Acre.
144 =	1				
1296 =	9 =	1			
39204 =	272¼ =	30¼ =	1		
1568160 =	10890 =	1210 =	40 =	1	
6272640 =	43560 =	4840 =	160 =	4 =	1

An imperial acre = 10 chains long by 1 chain broad, 640 imperial acres = 1 square mile, 36 square yards = 1 rood of mason's work. A square of thatching, slating, roofing, flooring, and partitioning are each = 100 square feet.

Avoirdupois Weight.

For all goods sold by weight, except gold, silver, and jewels.

Marked.		
drs.	16 drams	= 1 ounce.
oz.	16 ounces	= 1 pound.
lb.	28 pounds	= 1 quarter.
qrs.	4 quarters	= 1 hundredweight.
cwt.	20 hundredweight	= 1 ton, *t.*

14 lbs. = 1 stone, 8 stones or 112 lbs. = 1 cwt.

Drams.	Ounce.	Pound.	Qr.	Cwt.	Ton.
16 =	1				
256 =	16 =	1			
7168 =	448 =	28 =	1		
28672 =	1792 =	112 =	4 =	1	
573440 =	35840 =	2240 =	80 =	20 =	1

The pound avoirdupois is declared by statute equal to 7,000 troy grains.

Therefore	7000 lbs.	troy	=	5760 lbs.	avoir.
or	175 ,,	,,	=	144 ,,	,,
and	175 oz.	,,	=	192 oz.	,,

Hay and Straw.—36 lbs. of straw = 1 truss, 56 lbs. of old hay or 60 lbs. of new hay = 1 truss, and 36 trusses = 1 load; but straw and hay are generally sold by the stone or cwt. The hay of any year is considered new till the 1st of September.

Wool Weight.—7 lbs. = 1 clove, 2 cloves = 1 stone, 2 stones = 1 tod, 6½ tods or 182 lbs. = 1 wey, 2 weys or 364 lbs. = 1 sack, 12 sacks = 1 last; 20 lbs. = 1 score, 12 scores = 1 pack.

Solid or Cubic Measure.

In solid measure, length, breadth, and thickness are taken. Thus 1 foot, or 12 inches long × 12 inches broad × 12 inches deep = 1,728 solid inches = 1 solid or cubic foot, &c.

Solid In.		Solid Ft.		
1728	=	1		Solid Yd.
46656	=	27	=	1

40 solid feet of rough or 50 solid feet of hewn timber = 1 load.

Measures of Capacity

(*For both liquid and dry goods*).

Marked.		
gl.	4 gills	= 1 pint.
pts.	2 pints	= 1 quart.
qt.	4 quarts	= 1 gallon.
gall.	2 gallons	= 1 peck.
pks.	4 pecks	= 1 bushel.
bush.	8 bushels	= 1 quarter, *qr.*

Cub. Inch.	Gill.	Pint.	Quart.	Gallon.	Peck.	Bush.	Qr.
8·664 =	1						
34·659 =	4 =	1					
69·318 =	8 =	2 =	1				
277·274 =	32 =	8 =	4 =	1			
554·548 =	64 =	16 =	8 =	2 =	1		
2218·192 =	256 =	64 =	32 =	8 =	4 =	1	
17745·536 =	2048 =	512 =	256 =	64 =	32 =	8 =	1

42 gallons = 1 tierce, 63 gallons = 1 hogshead (hhd.), 84 gallons = 1 puncheon, 126 gallons = 1 pipe, 252 gallons = 1 tun.

The weights and measures in use should be imperial, and uniform in all districts, and apply to all commodities.

The variations of local weights and measures are so perplexing between corn and coals, hay and straw, wool and wheat, &c., that very few men of business even, if taken unprepared, can recollect the whole of them.

The 27th chapter of Magna Charta declares that the weights all over England are to be the same, but unnecessarily gives for different sorts of commodities two different sorts of weights—*troy* and *avoirdupois*.

The pound troy, consisting of 12 ounces, each ounce of 20 pennyweights, and each pennyweight of 32 *grains of wheat*, gathered in the middle of the ear, and well dried. The pennyweight has since been divided into 24 equal parts, called grains, and therefore weigh-1⅓ grains of wheat each. Dr. Hutton, however, estimated the grain troy at 1⅓ of wheat.

The avoirdupois, which from its more general utility is in greater use, has been computed by Dr. Hutton to contain 6,999½ grains troy; by Ferguson, 7,000; and by the academies of London and Paris, 7,004.

In dry measure, the following inconsistencies take place.

The *brass bushel* of Henry VII., found in the Ex-

chequer in 1688, contained 2,145 cubic inches; and it being known by experience that 1,728 cubic inches of wheat weigh $58\frac{17}{176}$ pounds troy, the above bushel will, therefore, contain 72lbs. troy of wheat, or 9 gallons weighing 8lbs. per gallon, and measuring $238\frac{1}{3}$ cubic inches each.

But, according to Greaves, in his "Origin of Weights and Measures," this same bushel, when filled with common spring water, and measured before the House of Commons in 1696, was found to contain $2{,}145\frac{6}{10}$ cubic inches, and the water weighed was equal to 1,131 ounces and 14 pennyweights, or 94 lbs 3 ounces and 14 pennyweights troy.

The Winchester gallon, measuring $272\frac{1}{4}$ cubic inches, contains 9 lbs. 13 ounces avoirdupois. But the Winchester bushel, legalised in 1697, measures $18\frac{1}{2}$ inches diameter and 8 inches in depth, and therefore contains $2{,}150\frac{42}{100}$ cubic inches, and its corresponding gallons should be $265\frac{8}{10}$ inches.

These are some of the inconsistencies in our present confused system of weights and measures.*

Measures of capacity are verified by ascertaining the weight of pure water they will contain at the temperature of maximum density (3·945° C., or 39·101° F.)

The verification of *measures of length* is made by means of what is called a comparator, a piece of mechanism upon which the bar to be verified may be placed, and determined in length by closely divided scales and verniers with microscopic observation, or by micrometers with finely divided screws, and large screw-heads divided on their circumference to one or more hundred parts.

* The Weights and Measures Act, which came into operation on January 1st, 1879, established the uniformity of weights and measures in the United Kingdom.

Metric Equivalents of Imperial Weights and Measures.

The units of the metric system are five, viz. :—

(1.) The *Metre*, the unit of length = 3·280899 feet = 39·37079 inches.

(2.) The *Are*, the unit of surface = the square of 10 metres = 119·60332 square yards.

(3.) The *Litre*, the unit of capacity = the cube of $\frac{1}{10}$ of a metre = 0·26418635 yards = 1·0567454 quarts = 2·1134908 pints.

(4.) The *Stere*, the unit of solidity = 1 cubic metre = 35·336636 cubic feet = 1·308764 cubic yards. This unit has fallen into general disuse.

(5.) The *Gramme*, the unit of weight = 15·43234874 grains troy.

Each unit has its decimal multiples and submultiples, *i.e.* weights and measures ten times larger or ten times smaller than the unit of the denomination preceding. These multiples and submultiples are indicated by prefixes placed before the names of the several fundamental units. The prefixes denoting multiples are derived from the Greek language, and are *deka*, ten; *hecto*, hundred; *kilo*, thousand; and *myria*, ten thousand. Those denoting submultiples are from the Latin, and are *deci*, tenth; *centi*, hundreth; and *milli*, thousandth.

The unit of itinerary measure is the *kilometre* = 0·62138 miles.

The unit of land measure is the *hectare* = 2·47114 acres.

The unit of commercial weight is the *kilogramme* = 2·2046425 lbs. avoirdupois.

To change *French grammes* into *lbs.* (avoir.) *Eng-*

lish, we have only to multiply the number of grammes by ·0022. To change *kilogrammes* into *cwts.*, multiply by ·1969. To change *lbs. English* into *kilogrammes French*, multiply by ·4535. To change *gallons* into *litres*, multiply by 4·543. To change *cubic inches* into *litres*, multiply by ·0163.

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